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TECHNICAL NOTE

No. 1435

STRESSES IN AND GENERAL INSTABILITY OF MONOCOQUE

CYLINDERS WITH CUTOUTS

V - CALCULATION OF THE STRESSES IN CYLINDERS

WITH SIDE CUTOUT

By N. J. Hoff and Bertram Klein

Polytechnic Institute of Brooklyn



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SUMMARY

Stresses were calculated by a numerical method in three reinforced monocoque cylinders subjected to pure bending. The cylinders were of circular cross section and were reinforced with 8 rings and either 8 or 16 stringers. There was a cutout on one side of each cylinder located symmetrically to the neutral plane and extending over 45° , 90° , or 135° . Satisfactory agreement was found between stresses calculated and those measured in part IV in the present series of investigations.

INTRODUCTION

In analytical investigations the reinforced monocoque cylinder is almost invariably assumed to be of constant section and reinforced with evenly spaced stringers and rings of constant cross-sectional properties. In reality, actual airplane structures often have openings for doors, windows, and so forth, and are reinforced locally near points of application of concentrated loads. It is believed that the stress problem of such nonuniform structures is best approached by numerical methods.

In a series of investigations carried out at the Polytechnic Institute of Brooklyn Aeronautical Laboratories an effort was made to apply Southwell's relaxation method (reference 1) to the calculation of the stresses in reinforced monocoque structures. Procedures were developed for reinforced flat and curved sheets (references 2 and 3) as well as for fuselage frames (references 4 and 5). Finally, numerical methods were used to determine the stresses in a reinforced monocoque cylinder having a symmetric cutout on the compression side (reference 6). The results obtained were in satisfactory agreement with experiments carried out earlier, which are described in reference 7. The present report deals with the problem of the stress distribution in a reinforced monocoque cylinder having a side cutout and subjected to pure bending. The results of the calculations are compared with the experiments described in reference 8.

In the first step of the procedure the structure is divided into elements, and the elastic properties of the elements are determined. In the present problem a sheet panel with its bordering segments of stringers and rings was chosen as the element of the reinforced monocoque cylinder. When the loads are applied to the cylinder, the corners of the panels undergo, in general, displacements in arbitrary directions. For the purposes of this calculation the displacements are resolved into axial (in the direction of the axis of the cylinder), tangential (in the direction of the tangent to the ring), and radial components (in the direction of the radius of the ring). At the same time, the corners are, in general, rotated about axes of arbitrary direction, and this rotation is resolved into rotations about the axial, tangential, and radial directions.

In the so-called unit problem it is assumed that the four corners of the panel are rigidly clamped to some imaginary rigid body to prevent both displacement and rotation. Then the clamps are released at one corner to permit displacement or rotation in one direction only, and a displacement (or rotation) of unit magnitude is undertaken in that particular direction. Next the reaction forces and moments caused by the displacement (or rotation) undertaken are calculated for all the four corners.

After all the unit problems of the structure are solved, the results are combined in what are termed the "operations tables." These tables are a systematic presentation of the reactions at all the corner points corresponding to unit displacements of the corner points. It is then required to find a combination of all the displacement (and rotation) components corresponding to zero resultant force and moment at each corner point at which no external load is applied and to force and moment resultants equal and opposite to the loads at the points of application of the external loads. According to Southwell's suggestion, this combination of displacements is found by systematic step-by-step approximations. At the Polytechnic Institute of Brooklyn Aeronautical Laboratories such solutions by step-by-step approximations have been established for reinforced panel problems (references 2 and 3), but when the same approach was tried for the case of monocoque cylinders having symmetric cut-outs and subjected to pure bending, the number of steps needed became almost prohibitive. On the other hand, the solution by matrix methods of the system of linear equations represented by the operations tables together with the applied loads was possible with a reasonable expenditure of work.

In the present report, the displacements are calculated from the operations tables by means of a slightly modified version of Crout's method of solving matrix equations. (See reference 9.) The number of unknowns is 34, 36, and 30 in the case of the cylinders having 45° , 90° , and 135° cutouts, respectively. The numerical part of the work was carried out on semiautomatic electric calculating machines, and 10 digits were kept throughout the calculations. As an approximate rule, it may

be stated that matrices of the kind encountered in this work can be solved by an experienced calculator at the rate of 2 hours for each unknown quantity. This estimate does not allow for mistakes.

Once the displacements are known, the stresses can be easily calculated with the aid of the solutions of the unit problems and elementary considerations. Complete numerical calculations were carried out for three cylinders of the experimental series described in reference 8. Satisfactory agreement was found between theory and experiment, as may be seen from the comparison shown in the figures of the present report.

The authors acknowledge their indebtedness to Mr. Bruno A. Boley for his help in the theoretical aspects of the problem and to Mr. John G. Pulos, who took part in the calculations. The work was carried out under the sponsorship and with the financial assistance of the National Advisory Committee for Aeronautics.

SYMBOLS

a	distance between adjacent rings
A	cross-sectional area of stringer augmented by its effective width of curved sheet
A, B, C, A', B', C'	rings of cylinder or quadrants of operations table
E	Young's modulus
G	shear modulus
L	length of ring segment between adjacent stringers
M	externally applied bending moment acting on cylinder
$\hat{m}$	ring influence coefficient
N	bending moment acting in plane of ring
Q	torque acting on rigid end ring
r	radius of monocoque cylinder
$\hat{m}, \hat{r}$	ring influence coefficients
R	radial shear force acting in plane of ring
t	thickness of sheet covering
$\hat{t}, \hat{r}, \hat{n}$	ring influence coefficients

T	tangential force acting in plane of ring
u	tangential displacement
v	radial displacement
w	rotation
X	force acting in axial direction
Y	vertical shear force acting on rigid ring
$\alpha_t, \alpha_r, \alpha_n$	coefficients used in calculation of forces and moments caused by shear flow existing in panel
$\Gamma = Gt/2$	
η	vertical downward translation of rigid ring
θ	rotation of rigid ring in its own plane
ξ	axial displacement
ω	rotation of rigid ring about its horizontal diameter
$\Omega = Gta/4L$	

STATEMENT OF PROBLEM AND ASSUMPTIONS

The three cylinders for which calculations were carried out are shown in figure 1. They are Cylinders 35, 39, and 40 of the present series and are described in reference 8. A number of structural changes were assumed for the purpose of calculations in order to decrease the work required for the solution of the problem.

Figure 2 shows the three cylinders in their modified forms. In reality, Cylinder 35 had 16 stringers, 7 of which were omitted in the simplified setup. The cross-sectional areas of the stringers eliminated, however, were distributed evenly between the adjacent stringers that were left in the structure. Similarly, two rings were omitted from the complete portions of the cylinder and the cross section of one was added to the adjacent ring bordering the cutout and the other to the rigid end ring of the cylinder. At the same time the length of the field extending from the cutout to the end ring was assumed to be 9.6429 inches. This field length is $1\frac{1}{2}$ times the actual ring spacing and thus it is an intermediate value between the true distance from cutout to end ring and the actual ring spacing. It was not considered advisable to use a

field length of three times the ring spacing in the calculations because long fields are weak in shear.

Cylinders 39 and 40 were built with only eight stringers. Consequently, changes in the structural arrangement were assumed only in connection with the rings. The changes were of the same nature as in the case of Cylinder 35.

As in previous work, the bending and torsional rigidities of the stringers were disregarded. The rings were considered resistant to bending in their own plane but weak in bending out of their plane as well as in torsion. The extensional and shearing rigidities of the rings were considered. The sheet panels were assumed to resist shear only, and the shear stresses were assumed to be distributed uniformly. The resistance of the sheet to extension and compression was taken approximately into account by adding an effective width of sheet to the stringers. In the present calculations the total width of the sheet was considered effective, since the stresses were calculated for small loads when the sheet is in a nonbuckled state. An effective width of sheet equal to the width of the ring was added to the ring when its cross-sectional properties were calculated. Because of these assumptions only the three displacement components as well as the rotation component about an axis parallel to the axis of the cylinder need be taken into consideration. Rotations about the tangential and radial axes are not resisted by either the stringer or the ring.

The vertical plane of a transverse section through the middle of the cylinder was regarded as a fixed reference plane relative to which the rigid end rings are tilted - and even twisted because of the asymmetric cutout - when the pure bending moments are applied to the end rings. The operations tables were set up for only one-quarter of the cylinder because the displacements in the four quarters are related by symmetry.

SETTING UP AND SOLVING THE OPERATIONS TABLES

A schematic arrangement showing the four quadrants of the operations tables for all three cylinders is given in table 1. As a rule, each entry in the operations tables (see, for instance, quadrant A, table 2) represents the magnitude and the sign of the generalized force, indicated at the left end of the row in which it appears, caused by the generalized unit displacement indicated at the top of the column. A generalized displacement is a displacement of the structure at a point in one of the directions of the axes, a rotation about one of these axes, or any combination of displacements and rotations of the structure at a group of points. A generalized force corresponding to a generalized displacement is the quantity - force, moment, or group of forces and moments - that gives the work done during the generalized displacement when multiplied by the generalized displacement. As was mentioned under STATEMENT OF PROBLEM

AND ASSUMPTIONS, the structure is considered to be rigidly clamped as regards every other generalized displacement when the effect of any one generalized unit displacement is sought.

The generalized forces in a reinforced monocoque cylinder caused by generalized unit displacements can be calculated when the solution of the so-called four-panel problem is known. The solution was given in reference 6. It is given in a slightly more convenient form in figures 3 to 6 of the present paper. These figures show the forces and moments at each of the nine corner points that are caused by generalized unit displacements of the middle point. The expressions are given in a form suitable for calculations even when each stringer and ring segment has a different but constant section and each panel a different but constant thickness. When a panel is in a buckled state, a reduced value should be used for its effective shear modulus G_{eff} . When a panel is absent, its shear modulus, or thickness, should be put equal to zero. The values of the shear flow-force coefficients α_t , α_r , and α_n , as well as those of the influence coefficients $\hat{t}t$, $\hat{t}r$, $\hat{t}n$, $\hat{r}r$, . . . , must be obtained from reference 5.

Figures 7 to 10 give the solution of the four-panel problem for the case in which the curvature is opposite to that shown in the preceding four figures. The calculations with which this report deals indicated the desirability of two such sets of diagrams in order to reduce the likelihood of numerical errors and errors of sign in the operations tables.

Because of the symmetry of both the structure and the loading with respect to the plane of a transverse section through the middle of the cylinder, displacements of corresponding points must be the same on rings A and A', B and B', and C and C'. (See fig. 2.) Moreover, the loading is antisymmetric with respect to the horizontal plane containing the axis of the cylinder. Hence, displacements of corresponding points on stringers 1 and 1', 2 and 2', and so forth, must be antisymmetric. Their absolute magnitudes are equal and their signs can be determined from the following rules, which take care of the peculiarities of the sign conventions adopted: axial and radial displacements are of opposite sign, tangential displacements and rotations are of the same sign on the upper and lower halves of the cylinder. These symmetry considerations permit a reduction in the number of displacement quantities to be entered in the operations tables. Of the total of $4 \times 48 = 192$ possible generalized basic displacements in the case of Cylinder 39, a total of 108 could be omitted outright; 36 more displacements were considered only indirectly, as is shown by means of the following two typical examples.

When point B4 - the point of intersection of ring B with stringer 4 - is moved in the positive axial direction, point B4' must be moved the same distance in the negative axial direction because of the antisymmetry.

This combination of displacements causes twice as much shear strain in the panel bounded by rings A and B and stringers 4 and 4' as would be caused by the displacement of point B4 alone. Consequently, the forces and moments appearing because of the shear at points A4 and B4 will be doubled.

When point B4 is moved in the positive tangential direction, point B4' also must be moved the same distance in the tangential direction. Consequently, the shear strain in the panel bounded by rings A and B and stringers 4 and 4' is again subjected to the double amount of shear strain just as in the case discussed previously. Moreover, segment 4-4' of ring B is rotated but not shortened, whereas in the case of a tangential motion of point B4 alone a shortening also would take place. Consequently, 48 independent displacement quantities remain to be entered in the operations tables. This number is further reduced because of the end conditions. In the experiment the end rings were heavy and were rigidly attached to heavy end plates. For this reason, in the theory the end rings were assumed perfectly rigid and points on the end rings were permitted to participate only in rigid body displacements. Thus $4 \times 4 = 16$ further individual generalized displacements are eliminated; and three rigid-body displacements are introduced — namely, a rotation ω about the horizontal diameter, a rotation θ about the axis of the cylinder, and a vertical translation η of the end ring. Hence 35 unknown quantities remain.

When the pure bending moment is applied to the rigid end plate, the distribution of the forces to the stringers is not known. Obviously, it cannot be assumed according to the customary linear law because of the cutout in the structure. For this reason a rotation ω of the end ring was specified rather than a bending moment, and the corresponding bending moment was calculated only after the forces in the stringers were determined from the operations table. Hence, the forces and moments corresponding in the operations tables to the specified rigid body displacement ω were known quantities and had to be considered as the load terms in the equations. They are given in the last column of quadrants B and D of table 2.

It will be noted that the last two rows in the operations tables are denoted $(1/2)Y$ (one-half the vertical shear force acting upon the end ring) and $(1/2)(Q/r)$ (one-half the torque acting upon the end ring divided by the radius). This choice of the quantities to be entered in the last two rows results in a symmetric operations table.

The linear equations represented by the operations tables were then solved by a slightly modified version of Crout's method. In other words, the set of 34 displacement quantities causing forces and moments at all the points equal and opposite to those given in the last column of the operations tables (which are due to the specified rotation ω) was determined. These forces and moments listed in the last column are designated RHS to indicate right-hand-side members. It should be noted that two of the displacement quantities listed are the remaining two

(unknown) rigid body displacements θ and η of the end ring. The generalized force corresponding to θ is a torque, that corresponding to η , a vertical shear force. Obviously, these two generalized displacements must be so chosen as to yield zero generalized forces when the external load applied to the cylinder is a pure bending moment. These two requirements are represented by the last two rows of the operations tables.

Similar considerations can be advanced in the case of the other two cylinders. The operations table of Cylinder 40 having the 135° cutout differs from table 2 only in quadrant D. This quadrant is given in table 3. In the case of Cylinder 35 all four quadrants are different. They are shown in table 4. In quadrant A of table 4 the columns of the tangential displacement and the rotation of point B1 correspond to two units each rather than to one. The doubling of these movements was undertaken in order to maintain the symmetry of the operations tables in spite of the assumptions regarding the simultaneous movements of points on the two sides of the horizontal plane of symmetry of the cylinder.

APPROXIMATE THEORY

Because of the great amount of work required for the solution of stress problems by the numerical method discussed, the possibility of using an approximate theory was investigated. The approximation amounted to neglecting all influences except that of the axial displacements. Physically the structure corresponding to the approximate theory would have rigid rings. Moreover, these rings would have to be supported in their own plane to provide reactions, since the shear forces and the torque acting upon the rings are not canceled in the approximate calculations.

The operations tables of the approximate theory are identical with those portions of the operations tables (tables 2 to 4) that involve only axial forces and displacements.

PRESENTATION AND DISCUSSION OF RESULTS

The displacements calculated for a rotation ω of the end ring amounting to 1×10^{-4} radian are presented in tables 5 to 7. The distortions of the rings corresponding to an applied bending moment of 20,000 inch-pounds are shown in figures 11 to 14.

The axial strains calculated from the displacements are plotted in figures 15 to 20, which also contain experimental results taken from reference 8 as well as calculated values corresponding to the approximate theory. The agreement between theory and experiment is

satisfactory. The approximate theory is also in reasonable agreement with experiment in the complete portions of the cylinders. In the cutout portions the values calculated by the approximate theory are even slightly closer to the experimental points than the values obtained from the complete theory. The displacements calculated by the approximate theory are listed in table 8.

Figures 21 and 22 show the shear stresses in the sheet of the complete portions of the cylinders and the maximum bending stresses in the rings bordering the cutout. The absolute values of these stresses are very small. Moreover, they decrease in an oscillatory manner from the region of the neutral axis of the cylinder on the cutout side toward the neutral-axis location on the opposite side.

The bending moment required to cause a rotation of 1×10^{-4} radian of the rigid end ring with respect to the transverse plane of symmetry is 5075.45, 7845.90, and 4511.04 inch-pounds in the case of the cylinders having 45° , 90° , and 135° cutouts, respectively. It should be remembered that the construction of the cylinder with the 90° cutout was different from that of the other two.

CONCLUSIONS

During the course of the calculation of the stresses in three reinforced monocoque cylinders with side cutout, carried out by means of a numerical procedure developed in part IV in the present series of investigations, the following principal observations were made:

1. The problem can be stated mathematically by means of a set of simultaneous linear equations represented by the operations tables and the external loads. The operations tables can be set up without difficulty if use is made of the solutions of the four-panel problem contained in the present report, together with the coefficients presented in the tables and graphs given in NACA TN No. 999.

2. The equations can be solved by Crout's method at the rate of approximately 2 hours for each unknown quantity. This estimate does not allow for errors.

3. The calculated values of the normal strain in the stringers were in satisfactory agreement with the strains measured in the experiments of part IV of the present series of investigations.

4. The shear stress in the sheet and the bending stress in the rings were found to be very small.

5. An approximate method which considers only the axial displacements and thus does not satisfy all the equilibrium conditions gave results reasonably close to those obtained by the complete method.

Polytechnic Institute of Brooklyn
Brooklyn N. Y., July 3, 1946

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TABLE 1 - SCHEMATIC ARRANGEMENT SHOWING THE FOUR
QUADRANTS OF THE OPERATIONS TABLES FOR ALL
THREE CYLINDERS

A	B
C	D

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TABLE 2.- OPERATIONS TABLE FOR CYLINDER 39 WITH 45° CUTOUT.

	E_{C4}	U_{C4}	V_{C4}	W_{C4}	E_{B4}	U_{B4}	V_{B4}	W_{B4}	E_{C3}	U_{C3}	V_{C3}	W_{C3}	E_{B3}	U_{B3}	V_{B3}	W_{B3}
X_{C4}	1179.6013728	11.577150	5.53227948	1.2284986	354.883868	11.577150	5.53227948	1.2284986	9.5766229	11.57715	1.84409316	1.2284986	9.5766229	11.57715	1.84409316	1.2284986
T_{C4}	11.577150	43.7125289	1.5108042	5.30127106	11.577150	41.986749	2.2293185	4.455384	11.57715	12.2824313	1.5108042	0.5133567	11.57715	13.995583	2.2293185	1.485128
R_{C4}	5.53227948	1.5108042	1.41480662	0.0769458	5.53227948	2.2293185	1.06530714	0.2385621	1.84409316	1.5108042	0.0769458	0.9769458	1.84409316	2.2293185	0.36510236	0.2365621
N_{C4}	1.2284986	5.30127106	0.0769458	2.03920102	1.2284986	4.455384	0.2365621	0.4777787	1.2284986	0.5133567	0.0769458	0.1499924	1.2284986	1.485128	0.2365621	0.1575929
X_{B4}	354.883868	11.577150	5.53227948	1.2284986	711.197279	0	0	0	9.5766229	11.57715	1.84409316	1.2284986	23.9415572	0	0	0
T_{B4}	11.577150	41.986749	2.2293185	4.455384	0	73.4294752	2.27850223	9.11741402	11.57715	13.995583	2.2293185	1.485128	0	19.0996684	2.27850223	0.5316707
R_{B4}	5.53227948	2.2293185	1.06530714	0.2365621	0	2.77850223	2.47451086	0.2327457	1.84409316	2.2293185	0.36510236	0.2365621	0	2.77850223	0.0395783	0.2327457
N_{B4}	1.2284986	4.455384	0.2365621	0.4777787	0	9.11741402	0.2327457	3.92060748	1.2284986	1.485128	0.2365621	0.1575929	0	0.5316707	0.2327457	0.3651577
X_{C3}	9.5766229	11.57715	1.84409316	1.2284986	9.5766229	11.57715	1.84409316	1.2284986	1170.027499	0	3.68818632	0	364.4705889	0	3.68818632	0
T_{C3}	11.57715	12.2824313	1.5108042	0.5133567	11.577150	13.995583	2.2293185	1.485128	0	31.4300976	0	4.78791436	0	27.991166	0	2.970256
R_{C3}	1.84409316	1.5108042	0.0769458	0.0769458	1.84409316	2.2293185	0.36510236	0.2365621	3.68818632	0	1.33583374	0	3.68818632	0	0.71020476	0
N_{C3}	1.2284986	0.5133567	0.0769458	0.1499924	1.2284986	1.485128	0.2365621	0.1575929	0	4.78791436	0	2.1001925	0	2.970256	0	0.3151858
X_{B3}	9.5766229	11.57715	1.84409316	1.2284986	23.9415572	0	0	0	364.4705889	0	3.68818632	0	887.2561707	0	0	0
T_{B3}	11.577150	13.995583	2.2293185	1.485128	0	19.0996684	2.27850223	0.5316707	0	27.991166	0	2.970256	0	53.3290068	0	8.36574332
R_{B3}	1.84409316	2.2293185	0.36510236	0.2365621	0	2.77850223	0.0395783	0.2327457	3.68818632	0	0.71020476	0	0	0	2.43403256	0
N_{B3}	1.2284986	1.485128	0.2365621	0.1575929	0	0.5316707	0.2327457	0.35251577	0	2.970256	0	0.3151858	0	0.5657433	0	4.2733226

QUADRANT A

— INDICATES NEGATIVE NUMBER

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X_{C2}									9.5766229	11.57715	1.84409316	1.2284986	9.5766229	11.57715	1.84409316	1.2284986
T_{C2}									11.57715	12.2824313	1.5108042	0.5133567	11.57715	13.995583	2.2293185	1.485128
R_{C2}									1.84409316	1.5108042	0.0769458	0.0769458	1.84409316	2.2293185	0.36510236	0.2365621
N_{C2}									1.2284986	0.5133567	0.0769458	0.1499924	1.2284986	1.485128	0.2365621	0.1575929
X_{B2}									9.5766229	11.57715	1.84409316	1.2284986	23.9415572	0	0	0
T_{B2}									11.57715	13.995583	2.2293185	1.485128	0	19.0996684	2.27850223	0.5316707
R_{B2}									1.84409316	2.2293185	0.36510236	0.2365621	0	2.77850223	0.0395783	0.2327457
N_{B2}									1.2284986	1.485128	0.2365621	0.1575929	0	0.5316707	0.2327457	0.3651577
X_{C1}																
T_{C1}																
R_{C1}																
N_{C1}																
X_{B1}																
T_{B1}																
R_{B1}																
N_{B1}																
$Y_{A/2}$																
$Q_{A/2}/T$																

QUADRANT C

— INDICATES NEGATIVE NUMBER

TABLE 2.- OPERATIONS TABLE FOR CYLINDER 39 WITH 45° CUTOUT - Concluded

	E _{C2}	U _{C2}	V _{C2}	W _{C2}	E _{B2}	U _{B2}	V _{B2}	W _{B2}	E _{C1}	U _{C1}	V _{C1}	W _{C1}	E _{B1}	U _{B1}	V _{B1}	W _{B1}	η _A	θ _A	-RHS
X _{C4}																			
T _{C4}																			
R _{C4}																			
N _{C4}																			
X _{B4}																	6.67800204	0	94.6507967
T _{B4}																	31.3726003	37.7175832	15.12627543
R _{B4}																	0.857401366	0	0.41338775
N _{B4}																	3.329103305	4.002365872	1.605110773
X _{C3}	9.5766229	11.577150	1.84409316	1.2284906	9.5766229	11.577150	1.84409316	1.2284906											
T _{C3}	11.577150	12.2824313	1.5108042	0.5133567	11.57715	13.995583	2.2293185	1.485128											
R _{C3}	1.84409316	1.5108042	0.07897280	0.0769458	1.84409316	2.2293185	0.355102380	0.2365621											
N _{C3}	1.2284906	0.5133567	0.0769458	0.1499024	1.2284906	1.485128	0.2365621	0.1575929											
X _{B3}	9.5766229	11.577150	1.84409316	1.2284906	23.9415572	0	0	0									16.124291458	0	228.507224068
T _{B3}	11.57715	13.995583	2.2293185	1.485128	0	19.8996684	2.278502230	0.5316707									12.905072772	37.7175832	6.28550843
R _{B3}	1.84409316	2.2293185	0.355102380	0.2365621	0	2.278502230	0.0385783	0.2327457									2.069950004	0	0.990018026
N _{B3}	1.2284906	1.485128	0.2365621	0.1575929	0	0.5316707	0.2327457	0.35751577									1.378659739	4.002365872	0.664556651

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X _{C2}	170.0247499	0	3.68816632	0	364.4705869	0	3.68816632	0	9.5766229	11.577150	1.84409316	1.2284906	9.5766229	11.57715	1.84409316	1.2284906			
T _{C2}	0	31.4300976	0	4.78791436	0	27.991166	0	2.970256	11.577150	12.2824313	1.5108042	0.5133567	11.57715	13.995583	2.2293185	1.485128			
R _{C2}	3.68816632	0	3.3383374	0	3.68816632	0	0.71020470	0	1.84409316	1.5108042	0.07897280	0.0769458	1.84409316	2.2293185	0.355102380	0.2365621			
N _{C2}	0	4.78791436	0	2.1891926	0	2.970256	0	0.3191858	1.2284906	0.5133567	0.0769458	0.1499024	1.2284906	1.485128	0.2365621	0.1575729			
X _{B2}	364.4705869	0	3.68816632	0	687.2561707	0	0	0	9.5766229	11.57715	1.84409316	1.2284906	23.9415572	0	0	0	16.124291458	0	228.507224068
T _{B2}	0	27.991166	0	2.970256	0	53.5298066	0	0.58574332	11.57715	13.995583	2.2293185	1.485128	0	19.8996684	2.278502230	0.5316707	12.905072772	37.7175832	6.28550843
R _{B2}	3.68816632	0	0.71020470	0	0	0	2.4348325	0	1.84409316	2.2293185	0.355102380	0.2365621	0	2.278502230	0.0385783	0.2327457	2.069950004	0	0.990018026
N _{B2}	0	2.970256	0	0.3191858	0	0.58574332	0	4.27332376	1.2284906	1.485128	0.2365621	0.1575929	0	0.5316707	0.2327457	0.35751577	1.378659739	4.002365872	0.664556651
X _{C1}	9.5766229	11.57715	1.84409316	1.2284906	9.5766229	11.577150	1.84409316	1.2284906	940.5670438	11.57715	1.84409316	1.2284906	900.7938174	11.57715	1.84409316	1.2284906			
T _{C1}	11.57715	12.2824313	1.5108042	0.5133567	11.577150	13.995583	2.2293185	1.485128	11.57715	15.7150488	2.93287392	3.3935710	11.577150	13.995583	2.2293185	1.485128			
R _{C1}	1.84409316	1.5108042	0.07897280	0.0769458	1.84409316	2.2293185	0.355102380	0.2365621	1.84409316	2.93287392	0.667916870	0.7020249	1.84409316	2.2293185	0.355102380	0.2365621			
N _{C1}	1.2284906	0.5133567	0.0769458	0.1499024	1.2284906	1.485128	0.2365621	0.1575929	1.2284906	2.393957100	0.7020249	0.0949963	1.2284906	1.485128	0.2365621	0.1575929			
X _{B1}	9.5766229	11.57715	1.84409316	1.2284906	23.9415572	0	0	0	900.7938174	11.57715	1.84409316	1.2284906	818.7507877	23.15430	3.68816632	4.589972	6.67800204	0	94.6507967
T _{B1}	11.577150	13.995583	2.2293185	1.485128	0	19.8996684	2.278502230	0.5316707	11.577150	13.995583	2.2293185	1.485128	23.15430	45.4383082	1.4370286	6.1471581	31.3726003	37.7175832	15.12627543
R _{B1}	1.84409316	2.2293185	0.355102380	0.2365621	0	2.278502230	0.0385783	0.2327457	1.84409316	2.2293185	0.355102380	0.2365621	3.6084864	1.4370286	1.7643081	0.8270158	0.857401366	0	0.41338775
N _{B1}	1.2284906	1.485128	0.2365621	0.1575929	0	0.5316707	0.2327457	0.35751577	1.2284906	1.485128	0.2365621	0.1575929	2.4569972	6.1471581	0.8270158	3.6084864	3.329103305	4.002365872	1.605110773
Y _{√2}																	77.3875188	0	34.90261462
Q _{W2} /r																	152.4713037	0	

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TABLE 2.- OPERATIONS TABLE FOR CYLINDER 39 WITH 45° CUTOUT.

	ξ_{C4}	u_{C4}	v_{C4}	w_{C4}	ξ_{B4}	u_{B4}	v_{B4}	w_{B4}	ξ_{C3}	u_{C3}	v_{C3}	w_{C3}	ξ_{B3}	u_{B3}	v_{B3}	w_{B3}
X_{C4}	179.6013728	11.577150	5.53227948	1.2284986	354.893986	11.577150	5.53227948	1.2284986	9.5766229	11.57715	1.84409316	1.2284986	9.5766229	11.57715	1.84409316	1.2284986
T_{C4}	11.577150	43.7125289	1.5108042	5.30127106	11.577150	41.906749	2.2293185	4.455384	11.57715	12.2824313	1.5108042	0.5133567	11.57715	13.995583	2.2293185	1.485128
R_{C4}	5.53227948	1.5108042	1.4180662	0.0769458	5.53227948	2.2293185	1.06530714	0.2365621	1.84409316	1.5108042	0.0769458	0.0769458	1.84409316	2.2293185	0.35510238	0.2365621
N_{C4}	1.2284986	5.30127106	0.0769458	2.1392002	1.2284986	4.455384	0.2365621	0.4777787	1.2284986	0.5133567	0.0769458	0.1409924	1.2284986	1.485128	0.2365621	0.1575929
X_{B4}	354.893986	11.577150	5.53227948	1.2284986	711.197279	0	0	0	9.5766229	11.57715	1.84409316	1.2284986	23.9415572	0	0	0
T_{B4}	11.577150	41.906749	2.2293185	4.455384	0	73.4294732	2.27850223	9.11741402	11.57715	13.995583	2.2293185	1.485128	0	19.8996684	2.27850223	0.5316707
R_{B4}	5.53227948	2.2293185	1.06530714	0.2365621	0	2.27850223	2.47451026	0.2327457	1.84409316	2.2293185	0.35510238	0.2365621	0	2.27850223	0.0395783	0.2327457
N_{B4}	1.2284986	4.455384	0.2365621	0.4777787	0	9.11741402	0.2327457	3.9208749	1.2284986	1.485128	0.2365621	0.1575929	0	0.5316707	0.2327457	0.35251577
X_{C3}	9.5766229	11.57715	1.84409316	1.2284986	9.5766229	11.57715	1.84409316	1.2284986	170.0247499	0	3.68816632	0	364.4785890	0	3.68816632	0
T_{C3}	11.57715	12.2824313	1.5108042	0.5133567	11.577150	13.995583	2.2293185	1.485128	0	31.4300970	0	4.78791436	0	27.991166	0	2.870256
R_{C3}	1.84409316	1.5108042	0.0769458	0.0769458	1.84409316	2.2293185	0.35510238	0.2365621	5.68816632	0	1.33583374	0	3.68816632	0	0.71029478	0
N_{C3}	1.2284986	0.5133567	0.0769458	0.1409924	1.2284986	1.485128	0.2365621	0.1575929	0	4.78791436	0	2.1891926	0	2.970256	0	0.3151658
X_{B3}	9.5766229	11.57715	1.84409316	1.2284986	23.9415572	0	0	0	364.4785890	0	3.68816632	0	867.2561707	0	0	0
T_{B3}	11.577150	13.995583	2.2293185	1.485128	0	19.8996684	2.27850223	0.5316707	0	27.991166	0	2.970256	0	53.5290668	0	8.56574332
R_{B3}	1.84409316	2.2293185	0.35510238	0.2365621	0	2.27850223	0.0395783	0.2327457	3.68816632	0	0.71029478	0	0	0	2.45403256	0
N_{B3}	1.2284986	1.485128	0.2365621	0.1575929	0	0.5316707	0.2327457	0.35251577	0	2.870256	0	0.3151658	0	8.5657433	0	4.27332326

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X_{C2}									9.5766229	11.57715	1.84409316	1.2284986	9.5766229	11.57715	1.84409316	1.2284986
T_{C2}									11.57715	12.2824313	1.5108042	0.5133567	11.57715	13.995583	2.2293185	1.485128
R_{C2}									1.84409316	1.5108042	0.0769458	0.0769458	1.84409316	2.2293185	0.35510238	0.2365621
N_{C2}									1.2284986	0.5133567	0.0769458	0.1409924	1.2284986	1.485128	0.2365621	0.1575929
X_{B2}									9.5766229	11.57715	1.84409316	1.2284986	23.9415572	0	0	0
T_{B2}									11.57715	13.995583	2.2293185	1.485128	0	19.8996684	2.27850223	0.5316707
R_{B2}									1.84409316	2.2293185	0.35510238	0.2365621	0	2.27850223	0.0395783	0.2327457
N_{B2}									1.2284986	1.485128	0.2365621	0.1575929	0	0.5316707	0.2327457	0.35251577
X_{C1}																
T_{C1}																
R_{C1}																
N_{C1}																
X_{B1}																
T_{B1}																
R_{B1}																
N_{B1}																
$Y_A/2$																
$(C_A/2)/r$																

QUADRANT C

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TABLE 2.- OPERATIONS TABLE FOR CYLINDER 39 WITH 45° CUTOUT. - Concluded

	E _{C2}	U _{C2}	V _{C2}	W _{C2}	E _{B2}	U _{B2}	V _{B2}	W _{B2}	E _{C1}	U _{C1}	V _{C1}	W _{C1}	E _{B1}	U _{B1}	V _{B1}	W _{B1}	η _A	ΔΓ	-RHS
X _{C4}																			
T _{C4}																			
R _{C4}																			
N _{C4}																			
X _{B4}																	6.67800204	0	94.6370087
T _{B4}																	31.3728003	37.7175802	15.1262763
R _{B4}																	0.857401366	0	0.41338775
N _{B4}																	3.329103305	4.002305972	1.605110773
X _{C3}	9.5766229	11.577150	1.84400316	2.284906	9.5766229	11.577150	1.84400316	2.284906											
T _{C3}	11.577150	12.2824313	1.3108042	0.5133567	11.577150	13.095583	2.2203185	1.485128											
R _{C3}	1.84400316	1.5108042	0.07697280	0.0769458	1.84400316	2.2203185	0.355102300	2.265621											
N _{C3}	1.2284906	0.5133567	0.0769458	0.1400024	1.2284906	1.485128	0.2365621	0.1575929											
X _{B3}	9.5766229	11.577150	1.84400316	2.284906	23.9415572	0	0	0									16.124281450	0	228.307224088
T _{B3}	11.577150	13.095583	2.2203185	1.485128	0	19.6998884	2.27850220	0.3316707									17.985077770	37.71758032	0.20550043
R _{B3}	1.84400316	2.2203185	0.355102300	2.265621	0	2.27850220	0.0385783	0.2327457									2.069590084	0	0.984000026
N _{B3}	1.2284906	1.485128	0.2365621	0.1575929	0	0.5316707	0.2327457	0.35251577									1.376997359	4.002305972	0.004856681

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	E _{C2}	U _{C2}	V _{C2}	W _{C2}	E _{B2}	U _{B2}	V _{B2}	W _{B2}	E _{C1}	U _{C1}	V _{C1}	W _{C1}	E _{B1}	U _{B1}	V _{B1}	W _{B1}	η _A	ΔΓ	-RHS
X _{C2}	170.0247899	0	3.68818632	0	364.4705889	0	3.68818632	0	9.5766229	11.577150	1.84400316	2.284906	9.5766229	11.577150	1.84400316	2.284906			
T _{C2}	0	31.4300876	0	4.78791436	0	27.998186	0	2.970256	11.577150	12.2824313	1.3108042	0.5133567	11.577150	13.095583	2.2203185	1.485128			
R _{C2}	3.68818632	0	1.33543374	0	3.68818632	0	0.71020476	0	1.84400316	1.5108042	0.07697280	0.0769458	1.84400316	2.2203185	0.355102300	2.265621			
N _{C2}	0	4.78791436	0	2.1891826	0	2.970256	0	0.3151858	1.2284906	0.5133567	0.0769458	0.1400024	1.2284906	1.485128	0.2365621	0.1575929			
X _{B2}	364.4705889	0	3.68818632	0	687.2361707	0	0	0	9.5766229	11.577150	1.84400316	2.284906	23.9415572	0	0	0	16.124281450	0	228.307224088
T _{B2}	0	27.998186	0	2.970256	0	33.5294086	0	1.58574332	11.577150	13.095583	2.2203185	1.485128	0	19.6998884	2.27850220	0.3316707	17.985077770	37.71758032	0.20550043
R _{B2}	3.68818632	0	0.71020476	0	0	0	2.4340325	0	1.84400316	2.2203185	0.355102300	2.265621	0	2.27850220	0.0385783	0.2327457	2.069590084	0	0.984000026
N _{B2}	0	2.970256	0	0.3151858	0	0.58574332	0	4.27332328	1.2284906	1.485128	0.2365621	0.1575929	0	0.5316707	0.2327457	0.35251577	1.376997359	4.002305972	0.004856681
X _{C1}	9.5766229	11.577150	1.84400316	2.284906	9.5766229	11.577150	1.84400316	2.284906	040.5670438	11.577150	1.84400316	2.284906	900.7938174	11.577150	1.84400316	2.284906			
T _{C1}	11.577150	12.2824313	1.3108042	0.5133567	11.577150	13.095583	2.2203185	1.485128	11.577150	15.7150408	2.92837292	3.9355714	11.577150	13.095583	2.2203185	1.485128			
R _{C1}	1.84400316	1.5108042	0.07697280	0.0769458	1.84400316	2.2203185	0.355102300	2.265621	1.84400316	2.03263330	0.67016670	7.020249	1.84400316	2.2203185	0.355102300	2.265621			
N _{C1}	1.2284906	0.5133567	0.0769458	0.1400024	1.2284906	1.485128	0.2365621	0.1575929	1.2284906	2.36395718	7.820349	1.094983	1.2284906	1.485128	0.2365621	0.1575929			
X _{B1}	9.5766229	11.577150	1.84400316	2.284906	23.9415572	0	0	0	900.7938174	11.577150	1.84400316	2.284906	618.7587877	23.15430	3.068189322	4.5061972	6.67800204	0	94.6370087
T _{B1}	11.577150	13.095583	2.2203185	1.485128	0	19.6998884	2.27850220	0.3316707	11.577150	13.095583	2.2203185	1.485128	23.15430	15.430082	4.5061972	6.67800204	31.3728003	37.71758032	15.1262763
R _{B1}	1.84400316	2.2203185	0.355102300	2.265621	0	2.27850220	0.0385783	0.2327457	1.84400316	2.2203185	0.355102300	2.265621	3.6681864	1.4370288	7.643081	0.6770158	0.857401366	0	0.41338775
N _{B1}	1.2284906	1.485128	0.2365621	0.1575929	0	0.5316707	0.2327457	0.35251577	1.2284906	1.485128	0.2365621	0.1575929	7.4669972	6.1471581	0.6270158	3.0050216	3.329103305	4.002305972	1.605110773
Y _{N/2}																	22.3875188	0	34.0001462
P _{A/2} /r																	182.4713037	0	

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TABLE 3.- OPERATIONS TABLE FOR CYLINDER 40 WITH 135° CUTOUT

	E_{C2}	U_{C2}	V_{C2}	W_{C2}	E_{B2}	U_{B2}	V_{B2}	W_{B2}	E_{B1}	U_{B1}	V_{B1}	W_{B1}	η_A	θ_A	-RHS
X_{C4}															
T_{C4}															
R_{C4}															
N_{C4}															
X_{B4}													6.678800284	0	94.05078167
T_{B4}													37.37288893	57.71758932	15.12627543
R_{B4}													0.857481384	0	0.413301779
N_{B4}													5.328103389	4.002368972	1.605110773
X_{C3}	9.5766229	11.577150	1.84408316	1.7284986	9.5766229	11.577150	1.84408316	1.7284986							
T_{C3}	11.577150	12.2824383	1.8108042	1.5133167	11.577150	13.005583	2.2293185	1.485128							
R_{C3}	1.84408316	1.5108042	0.07807288	0.0780458	1.84408316	2.2293185	0.35510238	0.2365621							
N_{C3}	1.2284986	0.5133567	0.0780458	0.1408024	1.2284986	1.485128	0.2365621	0.1575929							
X_{B3}	1.5766229	11.577150	1.84408316	1.7284986	13.0415572	0	0	0					18.124291450	0	278.507724868
T_{B3}	11.577150	13.005583	2.2293185	1.485128	0	13.005583	2.2293185	0.5316707					17.995972772	57.71758932	6.765288643
R_{B3}	1.84408316	2.2293185	0.35510238	0.2365621	0	2.2293185	0.35510238	0.2327457					2.058958004	0	0.998016825
N_{B3}	1.2284986	1.485128	0.2365621	0.1575929	0	0.5316707	0.2327457	0.35251577					1.376850730	4.002368972	0.64485865

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X_{C2}	040.5670438	11.57715	1.84408316	1.7284986	300.7535174	11.57715	1.84408316	1.7284986							
T_{C2}	11.57715	15.7150458	2.932637352	3.9395718	11.57715	13.905583	2.2293185	1.485128							
R_{C2}	1.84408316	2.932637352	0.0780458	0.0780458	1.84408316	2.2293185	0.35510238	0.2365621							
N_{C2}	1.2284986	2.9395718	0.0780458	0.0945063	1.2284986	1.485128	0.2365621	0.1575929							
X_{B2}	300.7535174	11.57715	1.84408316	1.7284986	300.7535174	11.57715	1.84408316	1.7284986	14.3640343	11.57715	1.84408316	1.7284986			
T_{B2}	11.57715	13.905583	2.2293185	1.485128	11.57715	13.905583	2.2293185	1.485128	11.57715	5.0040854	0.04918373	0.0534573			
R_{B2}	1.84408316	2.2293185	0.35510238	0.2365621	1.84408316	2.2293185	0.35510238	0.2365621	1.84408316	0.04918373	0.0534573	0.0534573			
N_{B2}	1.2284986	1.485128	0.2365621	0.1575929	1.2284986	1.485128	0.2365621	0.1575929	1.2284986	0.0534573	0.0534573	0.0534573			
X_{C1}					14.3640343	11.57715	1.84408316	1.7284986	14.3640343	11.57715	1.84408316	1.7284986			
T_{C1}					11.57715	5.0040854	0.04918373	0.0534573	11.57715	5.0040854	0.04918373	0.0534573			
R_{C1}					1.84408316	0.04918373	0.0534573	0.0534573	1.84408316	0.04918373	0.0534573	0.0534573			
N_{C1}					1.2284986	0.0534573	0.0534573	0.0534573	1.2284986	0.0534573	0.0534573	0.0534573			
X_{B1}															
T_{B1}															
R_{B1}															
N_{B1}															
$Y_A/2$															
$(Q_A/2)/\pi$															

QUADRANT D

— INDICATES NEGATIVE NUMBER

TABLE 4.- OPERATIONS TABLE FOR CYLINDER 35 WITH 90° CUTOUT

	$2u_{B1}$	$2w_{B2}$	ξ_{B2}	u_{B2}	v_{B2}	w_{B2}	ξ_{C2}	u_{C2}	v_{C2}	w_{C2}	ξ_{B3}	u_{B3}	v_{B3}	w_{B3}	ξ_{C3}	u_{C3}	v_{C3}	w_{C3}
T_{B1}	51.0772816	11.23087484	23.1543	11.8081700	0.88036746	1.9008146												
N_{B1}	11.23087484	7.91627449	2.4508972	1.9068146	0.93061254	1.02021734												
X_{B2}	23.1543	2.4508972	540.748891	11.57715	1.84408316	1.2284008	252.6825472	11.6701416	0.91784628	0.2985833	47.8831144	0	0	0	19.1532458	11.6701416	0.91784628	0.2985833
T_{B2}	11.8081700	1.9068146	11.57715	108.140457674	7.8452633	4.01336488	11.6701416	7.11085847	0.592470	0.18192727	0	72.64336253	15.94168084	10.85475158	11.6701416	7.11085847	0.592470	0.18192727
R_{B2}	0.09156746	0.93061254	1.84408316	14.78452633	4.60220514	1.77158578	0.91784628	0.592470	0.04308434	0.01430846	0	15.94168084	2.94740598	1.52788418	0.91784628	0.592470	0.04308434	0.01430846
N_{B2}	1.9068146	1.02021734	1.2284008	14.01336488	1.77158578	5.132718547	0.2985833	0.18192727	0.01430846	0.0046546905	0	15.94168084	1.52788418	0.562867183	0.2985833	0.18192727	0.01430846	0.0046546905
X_{C2}			252.6825472	11.6701416	0.91784628	0.2985833	834.6306746	11.6701416	0.91784628	0.2985833	19.1532458	11.6701416	0.91784628	0.2985833	19.1532458	11.6701416	0.91784628	0.2985833
T_{C2}			11.6701416	7.11085847	0.592470	0.18192727	11.6701416	40.37057847	0.931896	5.83314177	11.6701416	7.11085847	0.592470	0.18192727	11.6701416	13.9571537	0.77637	0.38705483
R_{C2}			0.91784628	0.592470	0.04308434	0.01430846	0.91784628	0.931896	1.67725134	4.3248446	0.91784628	0.592470	0.04308434	0.01430846	0.91784628	0.77637	1.46837226	0.76155734
N_{C2}			0.2985833	0.18192727	0.01430846	0.0046546905	0.2985833	5.83314177	4.3248446	5.7702089	0.2985833	0.18192727	0.01430846	0.0046546905	0.2985833	5.38794443	0.76155734	0.2985833
X_{B3}			47.8831144	0	0	0	19.1532458	11.6701416	0.91784628	0.2985833	834.6825477	0	0	0	431.2528581	0.082918	2.76193844	0.929163
T_{B3}			0	72.64336253	15.94168084	10.85475158	11.6701416	7.11085847	0.592470	0.18192727	0	72.64336253	15.94168084	10.85475158	0.8820916	21.10824147	1.670715	1.6670527
R_{B3}			0	15.94168084	2.94740598	1.52788418	0.91784628	0.592470	0.04308434	0.01430846	0	12.53520783	4.95730752	1.33501371	2.76193844	1.670715	1.39906670	0.7725332
N_{B3}			0	10.85475158	1.52788418	0.562867183	0.2985833	0.18192727	0.01430846	0.0046546905	0	15.94168084	1.52788418	0.562867183	0.2985833	1.6670527	1.27725382	1.022475805
X_{C3}			19.1532458	11.6701416	0.91784628	0.2985833	19.1532458	11.6701416	0.91784628	0.2985833	431.2528581	0.082918	2.76193844	0.929163	408.6780481	0.082918	1.76193844	0.929163
T_{C3}			11.6701416	7.11085847	0.592470	0.18192727	11.6701416	85.13857137	0.877637	5.36705483	0.082918	21.10824147	1.670715	1.6670527	0.082918	19.06777178	0.929163	0.929163
R_{C3}			0.91784628	0.592470	0.04308434	0.01430846	0.91784628	0.877637	1.46637226	0.76155734	2.76193844	1.670715	0.36906672	0.22225382	2.76193844	5.98226662	6431882	0.73046028
N_{C3}			0.2985833	0.18192727	0.01430846	0.0046546905	0.2985833	5.36705483	0.76155734	0.2985833	0.082918	21.10824147	0.22225382	0.162247930	0.082918	0.02703002	0.73046028	0.727198308

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X_{B4}											23.0415572	0	0	0	9.5768229	11.57715	1.84408316	1.2284008
T_{B4}											0	19.8008004	2.77850223	0.5316707	11.57715	13.985363	2.2284305	1.485128
R_{B4}											0	2.27850223	0.0385783	0.23274560	1.84408316	2.77850223	0.35510238	1.23666208
N_{B4}											0	0.5316707	0.23274560	0.35510238	1.2284008	1.485128	0.23658200	0.15758290
X_{C4}											9.5768229	11.57715	1.84408316	1.2284008	9.5768229	11.57715	1.84408316	1.2284008
T_{C4}											11.57715	13.985363	2.2284305	1.485128	11.57715	2.7824313	1.5180442	0.5133687
R_{C4}											1.84408316	2.2284305	0.35510238	0.23658200	1.84408316	1.5180442	0.07972882	0.0784582
N_{C4}											1.2284008	1.485128	0.23658200	0.15758290	1.2284008	0.5133687	0.0784582	0.1409824
X_{B5}																		
T_{B5}																		
R_{B5}																		
N_{B5}																		
X_{C5}																		
T_{C5}																		
R_{C5}																		
N_{C5}																		
$X_{/2}$																		

QUADRANT C

— INDICATES NEGATIVE NUMBER

TABLE 4.- OPERATIONS TABLE FOR CYLINDER 35 WITH 90° CUTOUT - Concluded

	E_{B4}	U_{B4}	V_{B4}	W_{B4}	E_{C4}	U_{C4}	V_{C4}	W_{C4}	E_{B5}	U_{B5}	V_{B5}	W_{B5}	E_{C5}	U_{C5}	V_{C5}	W_{C5}	η_A	η_T	-RHS
T_{B1}																	33.95778181	37.71750034	10.37296254
N_{B1}																	3.603389458	4.002305920	1.73735488
X_{B2}																	8.15043188	0	212.0082457
T_{B2}																	22.2257845	28.36320638	10.71804898
R_{B2}																	2.73182554	2.256305754	1.10500672
N_{B2}																	1.938910884	2.244373572	0.9334044
X_{C2}																			
T_{C2}																			
R_{C2}																			
N_{C2}																			
X_{B3}	23.9418572	0	0	0	9.6766229	11.57715	1.84408310	2.284906									12.9189707	0	277.0845833
T_{B3}	0	10.0906684	2.77850223	0.5316707	11.57715	13.095583	2.2293185	1.489128									5.24890371	28.36320638	2.52678671
R_{B3}	0	2.77850223	0.0395783	0.23274589	1.84408310	2.2293185	0.35510238	2.3500208									0.41266436	2.278395754	0.188984008
N_{B3}	0	0.5316707	0.23274589	0.3525577	1.2284906	1.489128	0.23556208	0.1579829									0.134242934	2.244373572	0.064787722
X_{C3}	9.5766229	11.57715	1.84408310	2.284906	9.6766229	11.57715	1.84408310	2.284906											
T_{C3}	11.57715	13.095583	2.2293185	1.489128	11.57715	12.7824313	1.5100042	0.5133567											
R_{C3}	1.84408310	2.2293185	0.35510238	0.23556208	1.84408310	1.5100042	0.07697280	0.07697280											
N_{C3}	1.2284906	1.489128	0.23556208	0.1579829	1.2284906	0.5133567	0.07697280	0.1490924											

QUADRANT B

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X_{B4}	1070.0009874	0	0	0	594.1870575	0	3.68818632	0	23.9418572	0	0	0	9.5766229	11.57715	1.84408310	2.284906	18.124291458	0	368.97564828
T_{B4}	0	10.0906684	0	0.58574632	0	27.994186	0	2.870286	0	19.0006684	2.77850223	0.5316707	11.57715	13.095583	2.2293185	1.489128	12.9189707	37.71750032	0.26250064
R_{B4}	0	0	2.43483256	0	3.68818632	0	0.71020470	0	0	2.77850223	0.0395783	0.23274589	1.84408310	2.2293185	0.35510238	0.23556208	2.08890004	9	0.99006020
N_{B4}	0	0.58574632	0	4.2732326	0	2.970286	0	0.5151096	0	0.5316707	0.23274589	0.3525577	1.2284906	1.489128	0.23556208	0.1579829	1.378829725	4.002305920	0.66485885
X_{C4}	594.1870575	0	3.68818632	0	1859.0841867	0	3.68818632	0	9.5766229	11.57715	1.84408310	2.284906	9.5766229	11.57715	1.84408310	2.284906			
T_{C4}	0	27.994186	0	2.970286	0	31.4300076	0	4.76594358	11.57715	13.095583	2.2293185	1.489128	11.57715	12.7824313	1.5100042	0.5133567			
R_{C4}	3.68818632	0	0.71020470	0	3.68818632	0	1.33565324	0	1.84408310	2.2293185	0.35510238	0.23556208	1.84408310	1.5100042	0.07697280	0.07697280			
N_{C4}	0	2.970286	0	0.5151096	0	4.76594358	0	2.1891025	1.2284906	1.489128	0.23556208	0.1579829	1.2284906	0.5133567	0.07697280	0.1490924			
X_{B5}	23.9418572	0	0	0	9.5766229	11.57715	1.84408310	2.284906	1004.0085246	0	0	0	9.5766229	11.57715	1.84408310	2.284906	0.67890000	0	153.24809132
T_{B5}	0	10.0906684	2.77850223	0.5316707	11.57715	13.095583	2.2293185	1.489128	0	13.4294752	2.77850223	0.5316707	11.57715	41.906749	2.2293185	1.489128	31.57280083	37.71750032	15.128754
R_{B5}	0	2.77850223	0.0395783	0.23274589	1.84408310	2.2293185	0.35510238	2.3500208	0	2.77850223	2.47458086	0.23274589	5.53277948	2.2293185	1.065307140	0.23556208	0.05740306	0	0.413380773
N_{B5}	0	0.5316707	0.23274589	0.3525577	1.2284906	1.489128	0.23556208	0.1579829	0	0.5316707	0.23274589	0.3525577	1.2284906	4.485384	0.23556208	0.1579829	3.39103388	4.002305920	1.05110772
X_{C5}	9.5766229	11.57715	1.84408310	2.284906	9.5766229	11.57715	1.84408310	2.284906	594.1870575	11.57715	13.095583	2.2293185	1.489128	11.57715	12.7824313	1.5100042			
T_{C5}	11.57715	13.095583	2.2293185	1.489128	11.57715	12.7824313	1.5100042	0.5133567	11.57715	41.906749	2.2293185	1.489128	11.57715	45.705289	1.5100042	0.5133567			
R_{C5}	1.84408310	2.2293185	0.35510238	0.23556208	1.84408310	1.5100042	0.07697280	0.07697280	5.53277948	2.2293185	1.06530714	0.23556208	5.53277948	1.5100042	0.41400000	0.07697280			
N_{C5}	1.2284906	1.489128	0.23556208	0.1579829	1.2284906	0.5133567	0.07697280	0.1490924	1.2284906	4.485384	0.23556208	0.1579829	1.2284906	5.30127003	0.07697280	0.1490924			
$Y_A/2$	18.124291458	2.8881775	2.8881775	2.8881775	1.78898735				6.67890000	31.57280083	0.65740306	0.23556208		1.2284906	5.30127003	0.07697280	0.1490924		
$(Q_A/2)/A$	0	37.71750032	0	4.002305920					0	37.71750032	0	4.002305920					45.0037008	0	70.20251454

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— INDICATES NEGATIVE NUMBER

TABLE 5.— DEFLECTIONS AND ROTATIONS FOR
CYLINDER WITH 45° CUTOUT

[$M = 5075.45$ in.-lb; $\eta = 0.934679$;
 $\theta_r = -0.182954$; unit for values
is 1×10^{-3} in. or radian]

		Ring A	Ring B	Ring C
Stringer 1*	ξ u v w	-0.382683 .680577 .357683 -.018295	-0.211436 .000999 .000110 -.013137	45° cutout
Stringer 1	ξ u v w	.382683 .680577 -.357683 -.018295	.211436 .000999 -.000110 -.013137	0.070479 -.145643 -.050187 -.012965
Stringer 2	ξ u v w	.923880 .174729 -.863531 -.018295	.462027 -.086238 -.232744 -.028647	.154009 -.176519 -.024713 -.013301
Stringer 3	ξ u v w	.923880 -.540637 -.863531 -.018295	.461874 -.269938 -.185258 -.010892	.153958 -.183646 .002972 -.016637
Stringer 4	ξ u v w	.382683 -1.046485 -.357683 -.018295	.191394 -.376640 -.086306 -.020508	.063798 -.179948 .003054 -.018817
Stringer 4*	ξ u v w	-.382683 -1.046485 .357683 -.018295	-.191394 -.376640 .086306 -.020508	-.063798 -.179948 -.003054 -.018817

TABLE 6.— DEFLECTIONS AND ROTATIONS FOR
CYLINDER WITH 90° CUTOUT

[$M = 7845.90$ in.-lb; $\eta = 0.877023$;
 $\theta_r = 0.128525$; unit for values
is 1×10^{-3} in. or radian]

		Ring A	Ring B	Ring C
Stringer 2 ^a	ξ u v w	-0.707107 .748674 .620149 .012853	-0.443778 .316729 -.139502 -.081652	90° cutout
Stringer 1	ξ u v w	0 1.005548 0 .012853	0 .133452 0 .123108	
Stringer 2	ξ u v w	.707107 .748674 -.620149 .012853	.443778 .316729 .139502 -.081652	0.147921 .396281 -.614729 .095219
Stringer 3	ξ u v w	.923880 .464147 -.810264 .012853	.462609 .275678 -.327575 -.055475	.154210 .203256 -.356365 .093722
Stringer 4	ξ u v w	.923880 -.207097 -.810264 .012853	.461645 .034594 -.088412 .057352	.153875 .128796 .092635 .032331
Stringer 5	ξ u v w	.382683 -.681739 -.335622 .012853	.191577 -.003785 -.088206 .000093	.063867 .189788 .020932 .006447
Stringer 5 ^a	ξ u v w	-.382683 -.681439 .335622 .012853	-.191577 -.003785 .088206 .000093	-.063867 .189788 -.020932 .006447

TABLE 7.— DEFLECTIONS AND ROTATIONS FOR
CYLINDER WITH 135° CUTOUT

[$M = 4511.04$ in.-lb; $\eta = 0.780683$;
 $\theta_r = -0.02370$; unit for values
is 1×10^{-3} in. or radian]

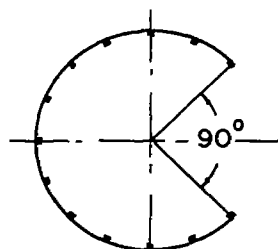
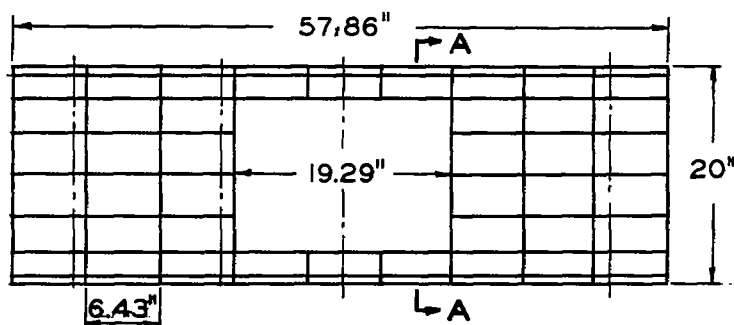
		Ring A	Ring B	Ring C
Stringer 2*	ξ u v w	-.923880 .275052 .721257 -.002370	-.511401 .142873 .276732 -.093973	135° cutout
Stringer 1*	ξ u v w	-.382683 .697557 .298752 -.002370	-.381188 -.095065 -.652765 .045670	
Stringer 1	ξ u v w	.382683 .697557 -.298752 -.002370	.381188 -.095065 .652765 .045670	
Stringer 2	ξ u v w	.923880 .275052 -.721257 -.002370	.511401 .142873 -.276732 -.093973	0.170466 .057603 -.099144 .025017
Stringer 3	ξ u v w	.923880 -.322452 -.721257 -.002370	.461519 -.075011 -.035912 .054762	.153841 .043337 .061874 .022328
Stringer 4	ξ u v w	.382683 -.744957 -.298752 -.002370	.191637 -.067597 -.049547 -.015939	.063876 .120387 .088380 -.003614
Stringer 4*	ξ u v w	-.382683 -.744957 .298752 -.002370	-.191637 -.067597 .049547 -.015939	-.063876 .120387 -.088380 -.003614
Stringer 3*				

TABLE 8.— AXIAL DEFLECTIONS FOR APPROXIMATE SOLUTIONS

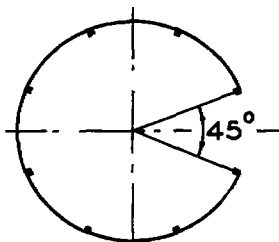
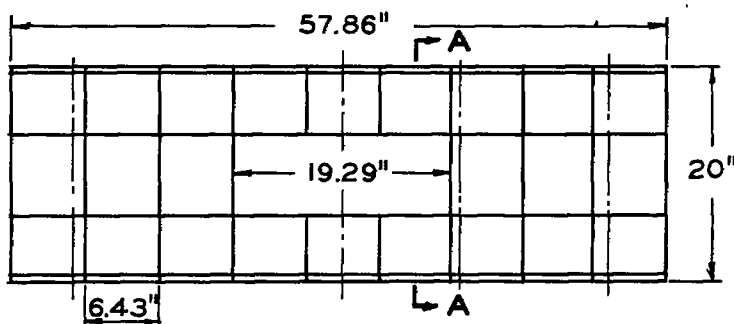
[Unit for values is 1×10^{-3} in.]

Angle of cutout	Stringer	Ring A	Ring B	Ring C	Average moment (in.-lb)
45°	1	0.382683	0.206436	0.071479	
	2	.923880	.432027	.0142009	
	3	.923880	.431874	.140958	
	4	.382683	.179394	.058798	
M(in.-lb)		5392	4788	4696	4959
90°	1	0	0	0	
	2	.707107	.426278	.143121	
	3	.923880	.450609	.149910	
	4	.923880	.443144	.146275	
	5	.382683	.183577	.060567	
M(in.-lb)		8155	7586	7533	7758
135°	1	.382683	.341188	— — — —	
	2	.923880	.484401	.160466	
	3	.923880	.433019	.142341	
	4	.382683	.179637	.059176	
M(in.-lb)		4875	4272	4206	4451

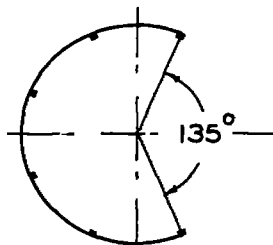
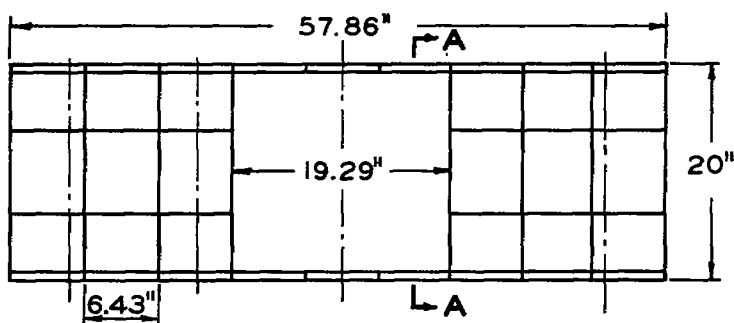
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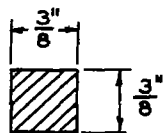
SECTION A-A
PIBAL CYL. NO. 35
16 STRINGERS



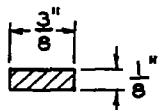
SECTION A-A
PIBAL CYL. NO. 39
8 STRINGERS



SECTION A-A
PIBAL CYL. NO. 40
8 STRINGERS



STRINGER SECTION



RING SECTION

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24S-T aluminum alloy

24S-T Alclad sheet

THICKNESS = 0.012"

----- Center line of strain gages

Figure 1.- Actual monocoque cylinders. (Described in reference 8.)

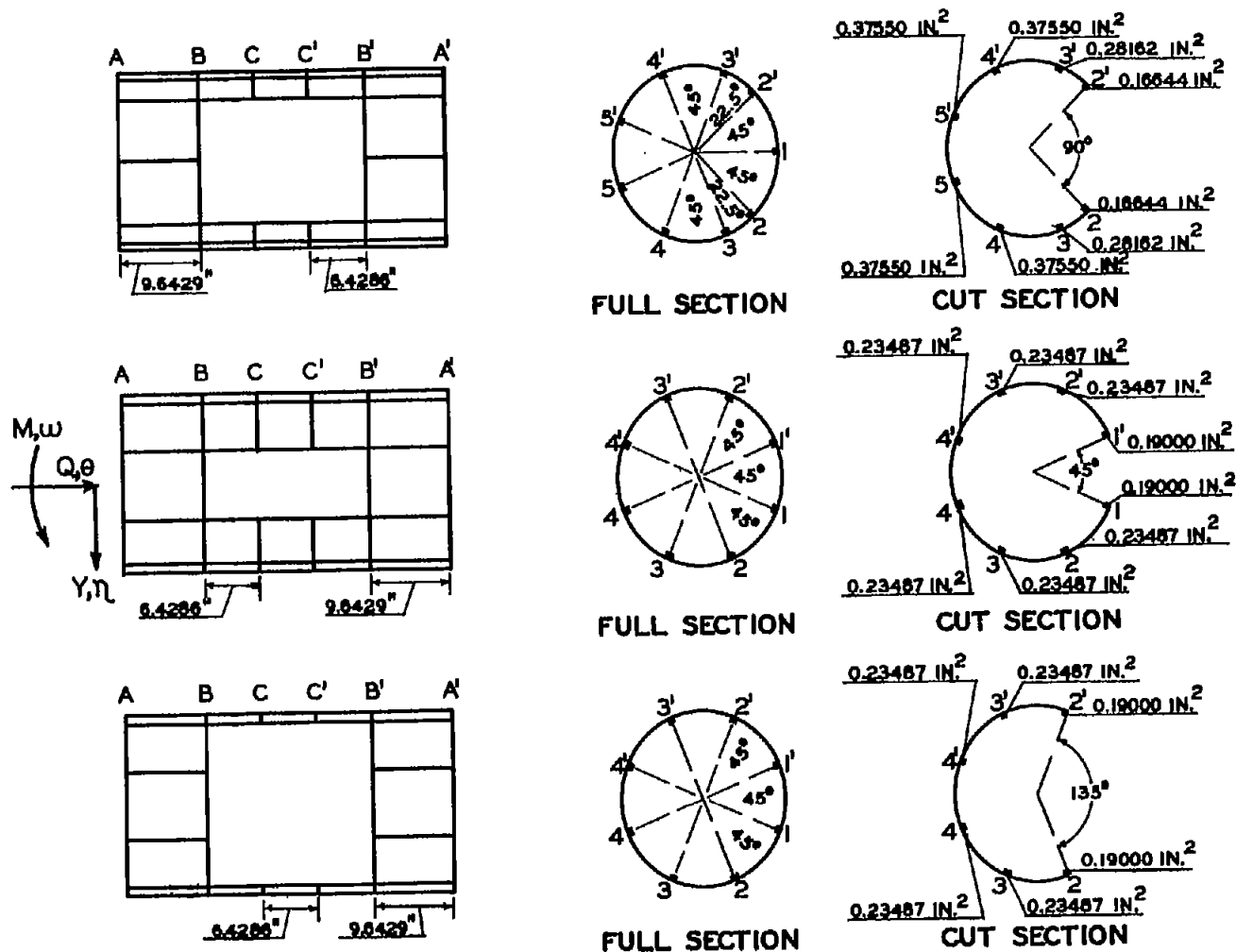


Figure 2.- Modified monocoque cylinders.

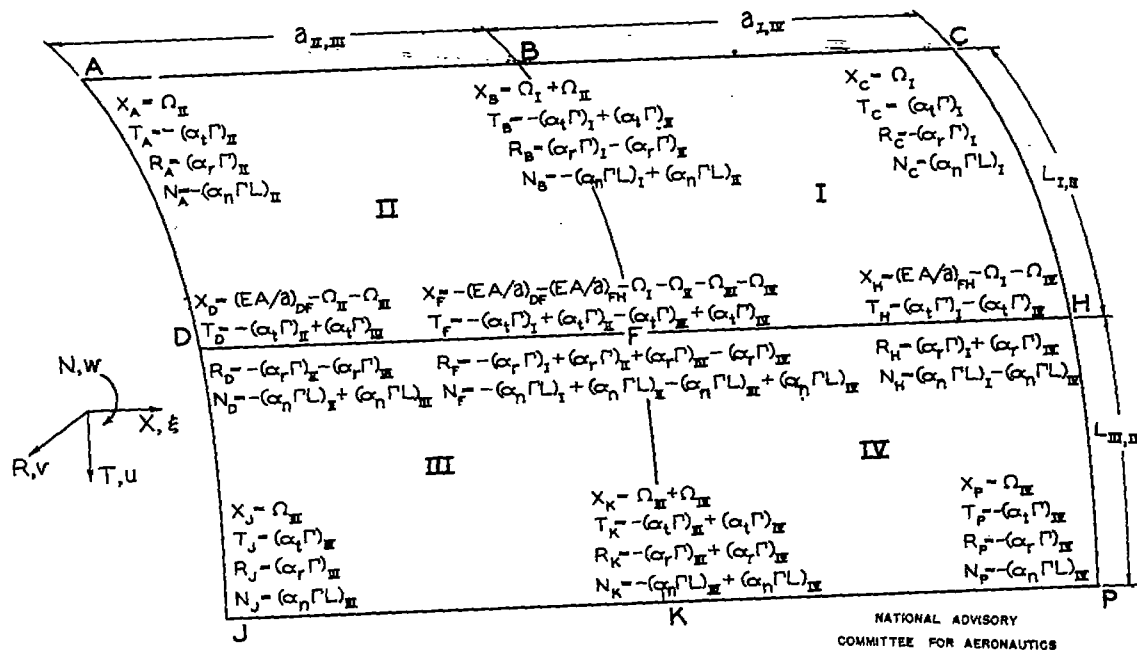


Figure 3.- Effect of unit axial displacement of F. Forces and moments acting on constraints.

$$\Gamma = \frac{Gt}{2}; \quad \Omega = \frac{Gta}{4L}$$

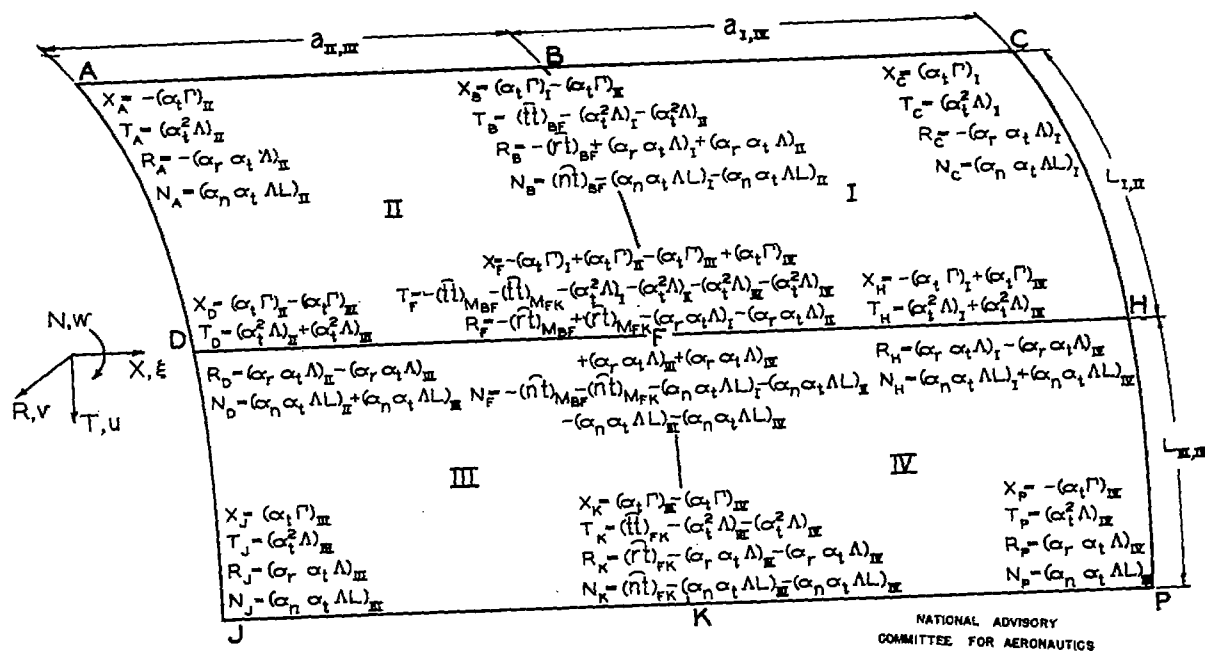


Figure 4.- Effect of unit tangential displacement of F. Forces and moments acting on constraints.

$$\Gamma = \frac{Gt}{2}; \quad \Lambda = \frac{GtL}{8}$$

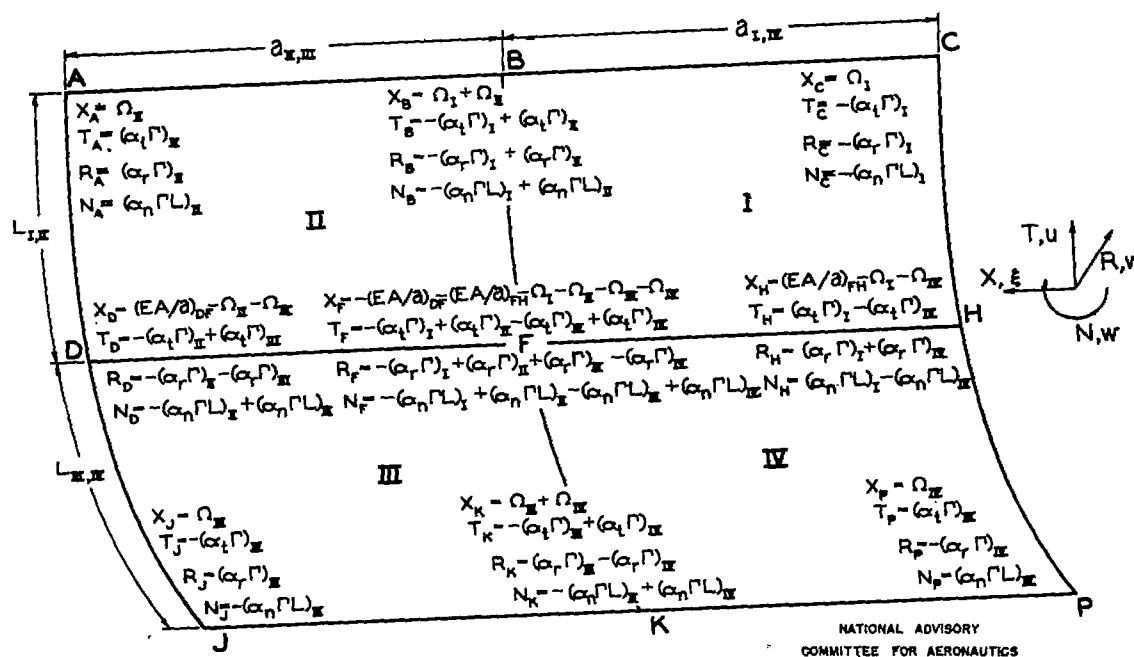


Figure 7.- Effect of unit axial displacement of F. Forces and moments acting on constraints.
 $r = \frac{Gt}{2}$; $n = \frac{Gt\alpha}{4L}$. (Curvature opposite that in figs. 3 to 6.)

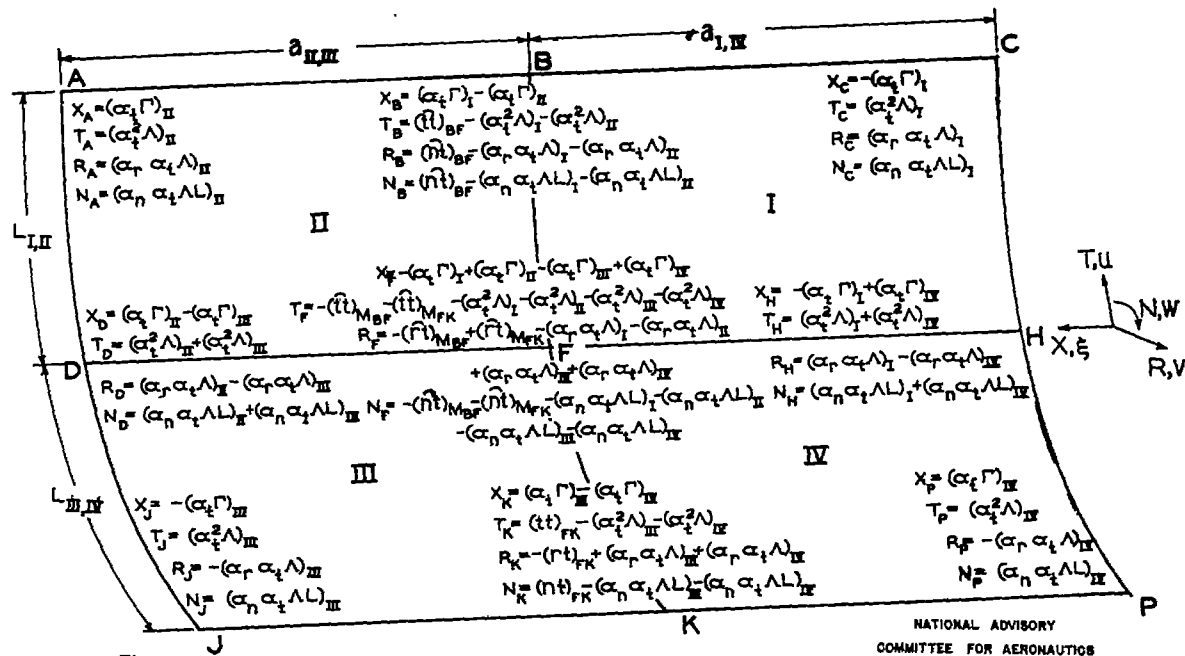


Figure 8.- Effect of unit tangential displacement of F. Forces and moments acting on constraints.
 $r = \frac{Gt}{2}$; $\Lambda = \frac{GtL}{\alpha}$. (Curvature opposite that in figs. 3 to 6.)

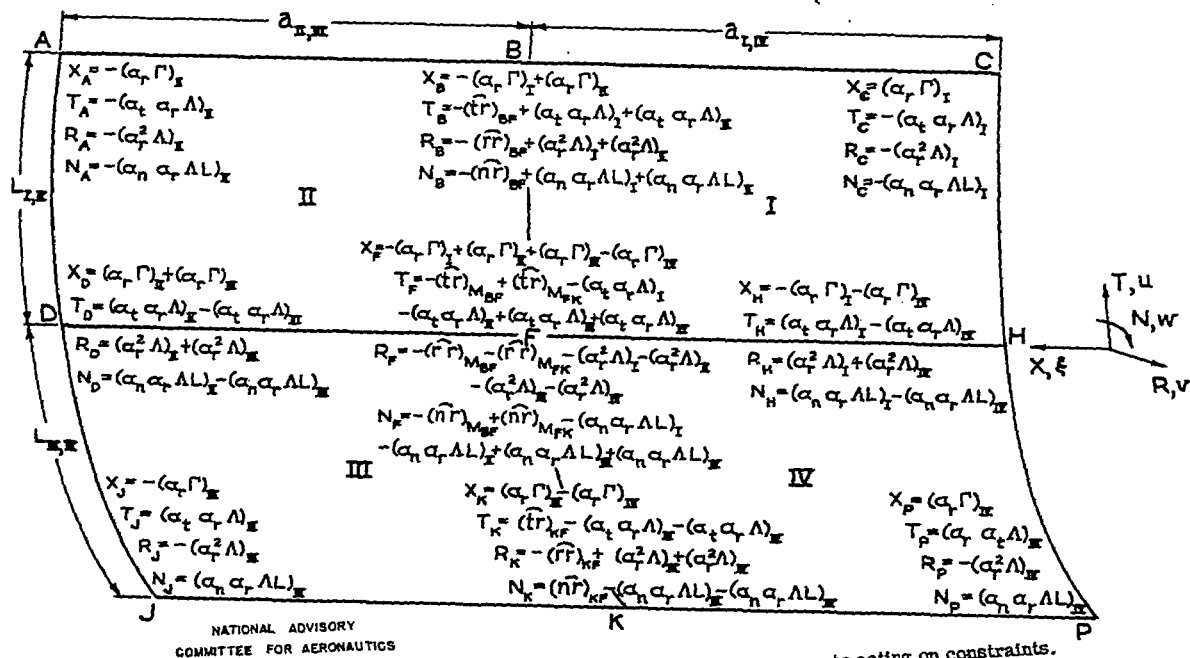


Figure 9.- Effect of unit radial displacement of F. Forces and moments acting on constraints.
 $\Gamma = \frac{Gt}{2}$; $\Lambda = \frac{GtL}{8}$. (Curvature opposite that in figs. 3 to 6.)

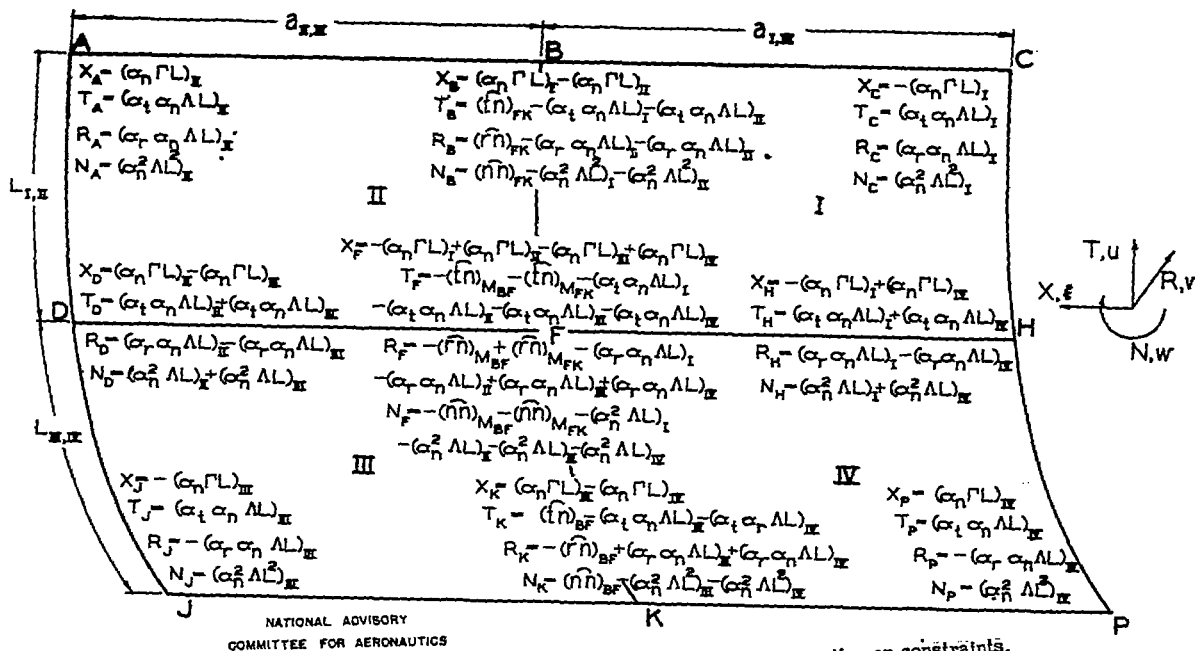


Figure 10.- Effect of unit rotation of F. Forces and moments acting on constraints.
 $\Gamma = \frac{Gt}{2}$; $\Lambda = \frac{GtL}{8}$. (Curvature opposite that in figs. 3 to 6.)

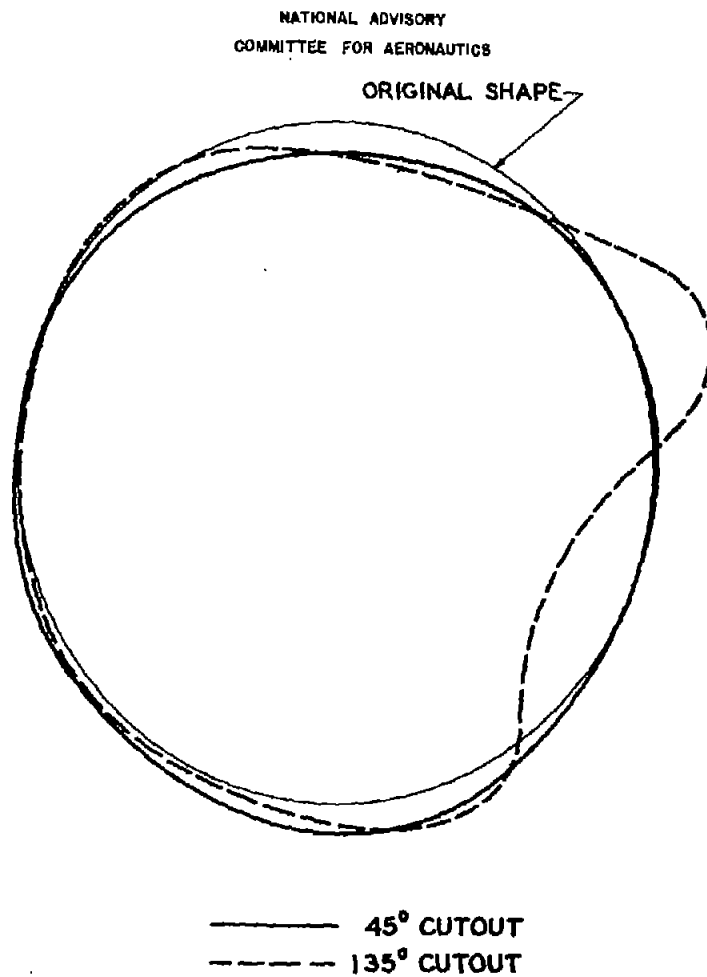


Figure 11.- Deflected shape of full rings in their own planes.

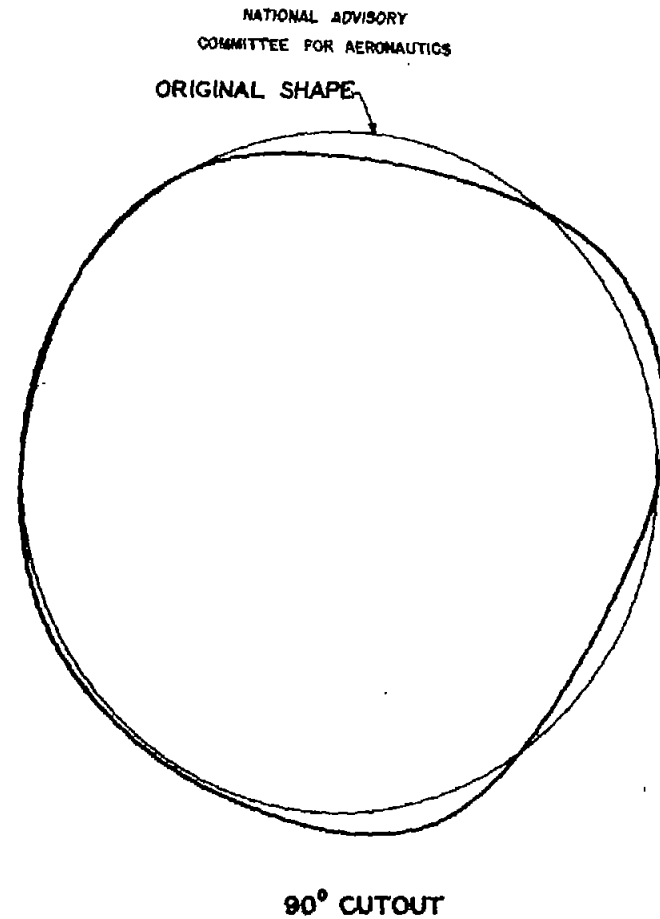


Figure 12.- Deflected shape of full ring in its own plane.

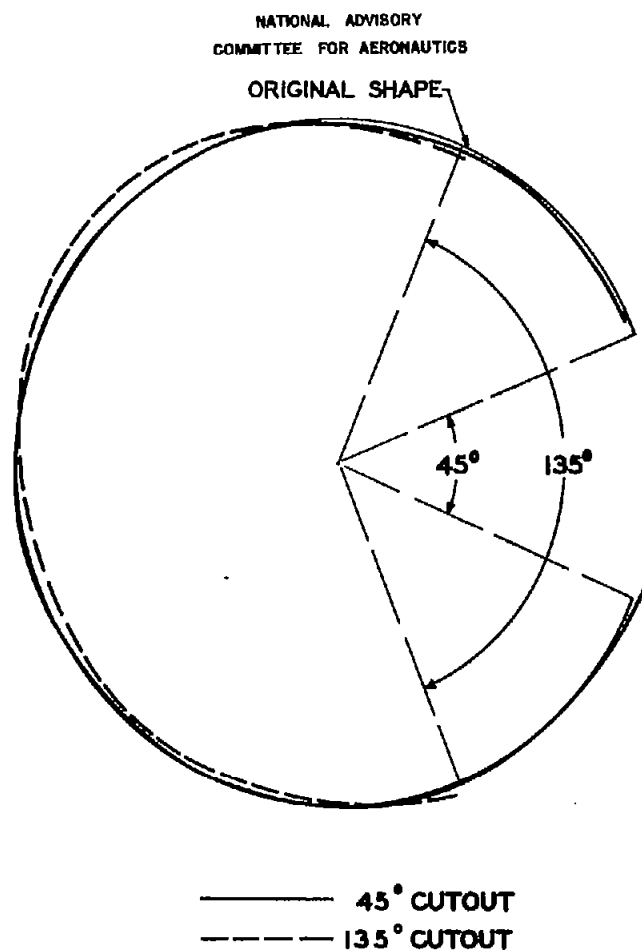


Figure 13.- Deflected shape of cut rings in their own planes.

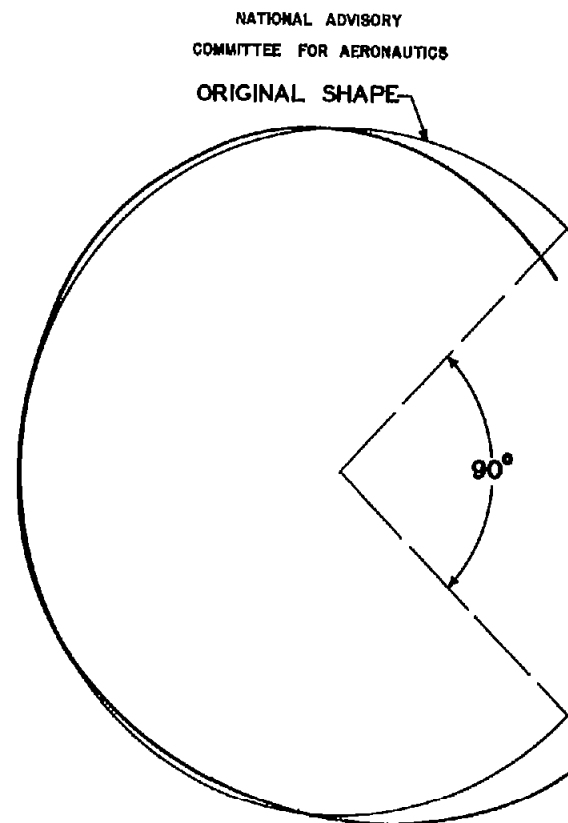


Figure 14.- Deflected shape of cut ring in its own plane.

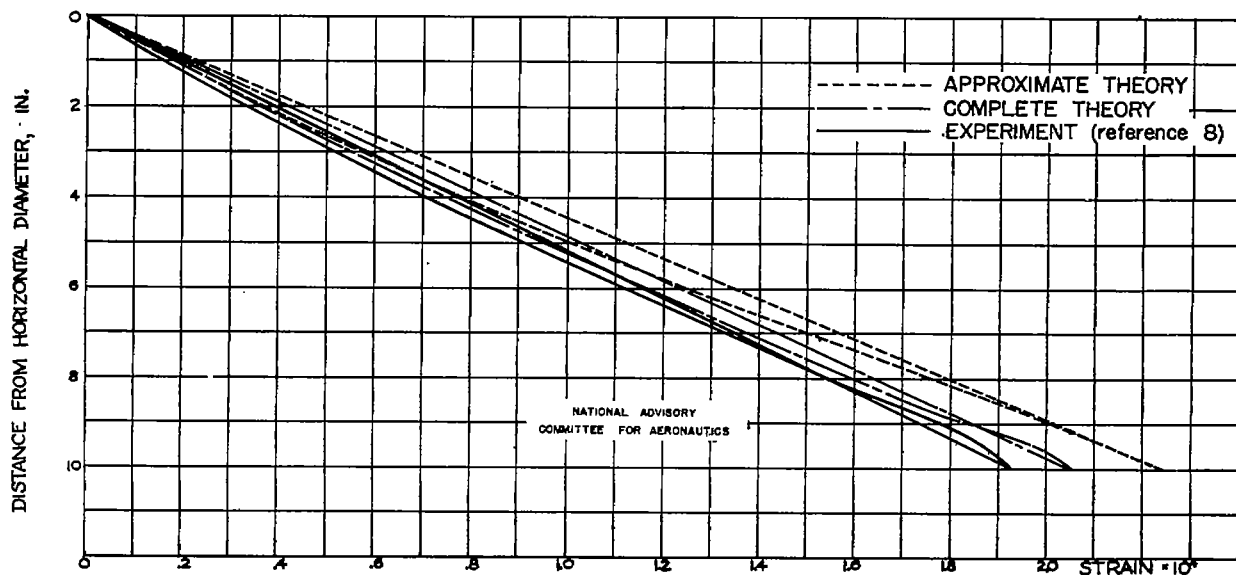


Figure 16.- Comparison of normal strain. Full section, 8 stringers, 45° cutout, $M = 20,000$ in.-lb.

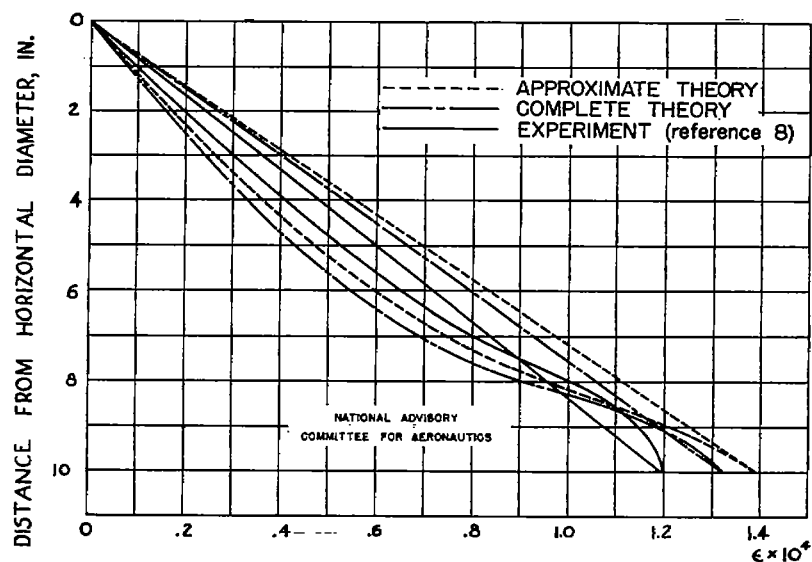


Figure 16.- Comparison of variation of normal strain. Full section, 16 stringers, 90° cutout, $M = 20,000$ in.-lb.

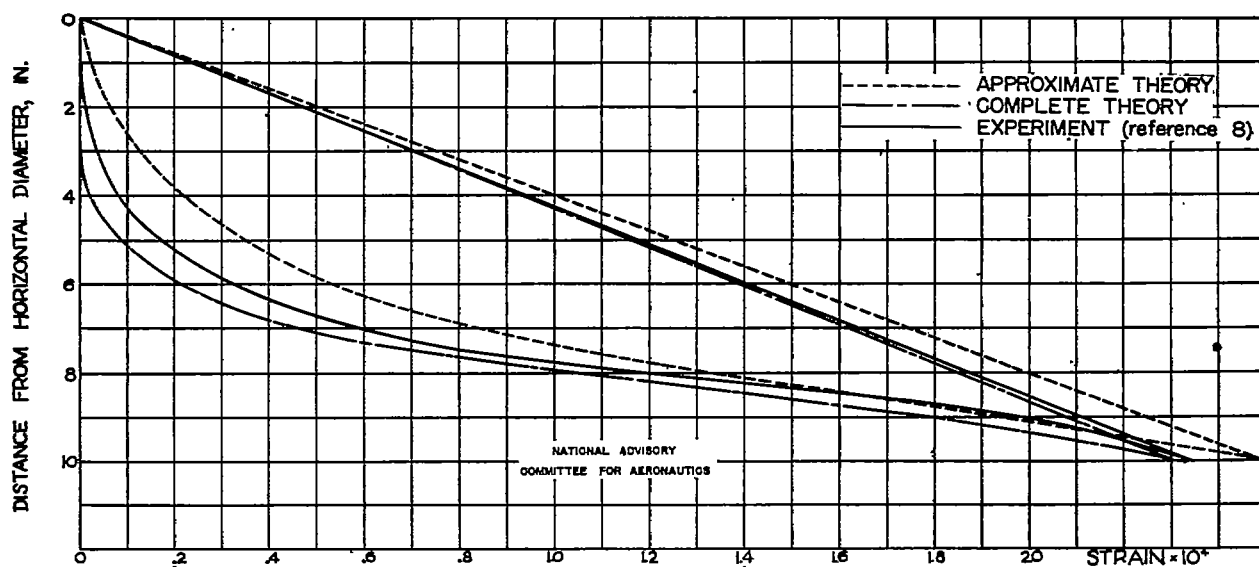


Figure 17.- Comparison of normal strain. Full section, 8 stringers, 135° cutout, $M = 20,000$ in.-lb.

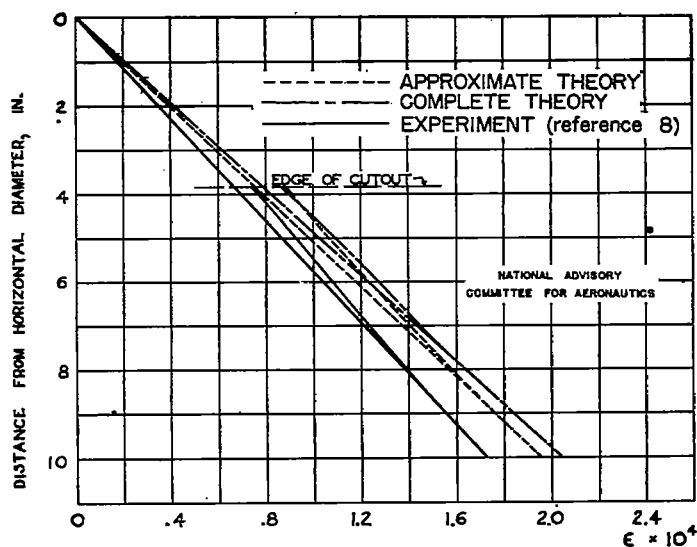


Figure 18.- Comparison of variation of normal strain. Cutout section, 8 stringers, 45° cutout, $M = 20,000$ in.-lb.

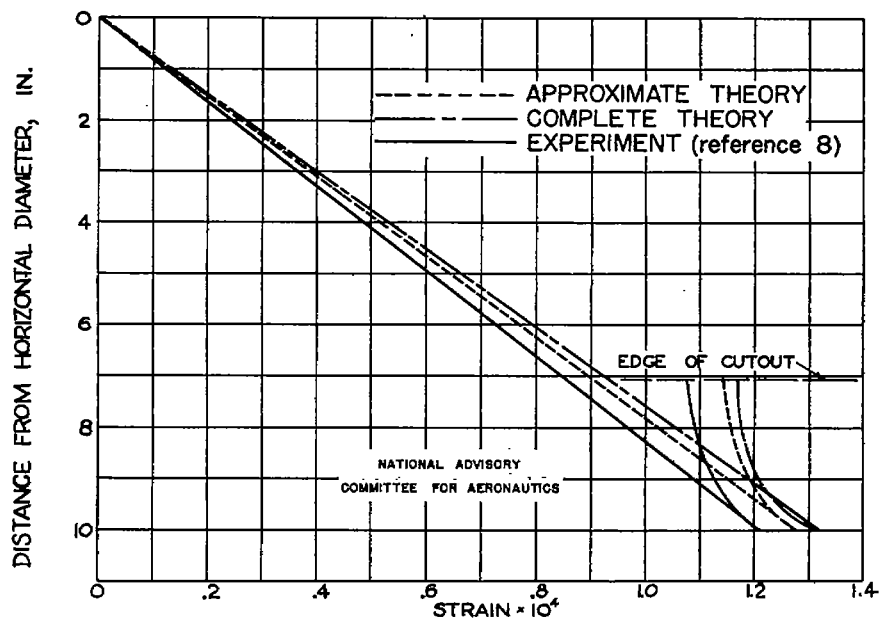


Figure 19.- Comparison of variation of normal strain. Cutout section, 18 stringers, 90° cutout, $M = 20,000$ in-lb.

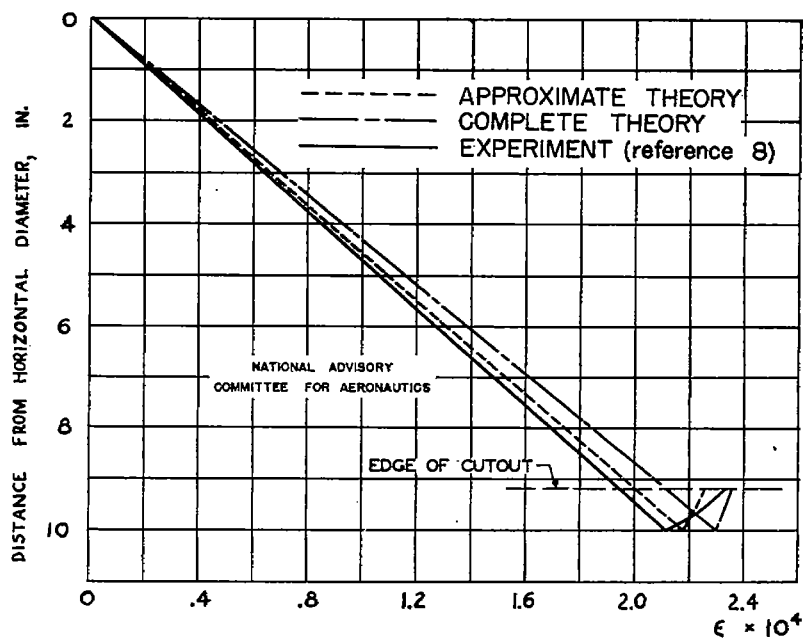
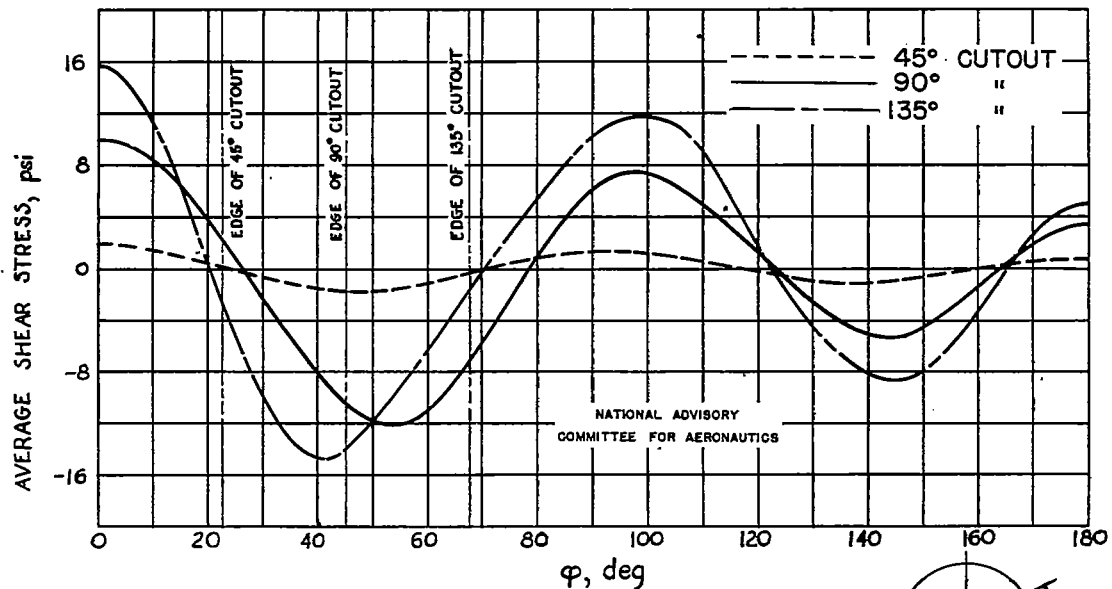
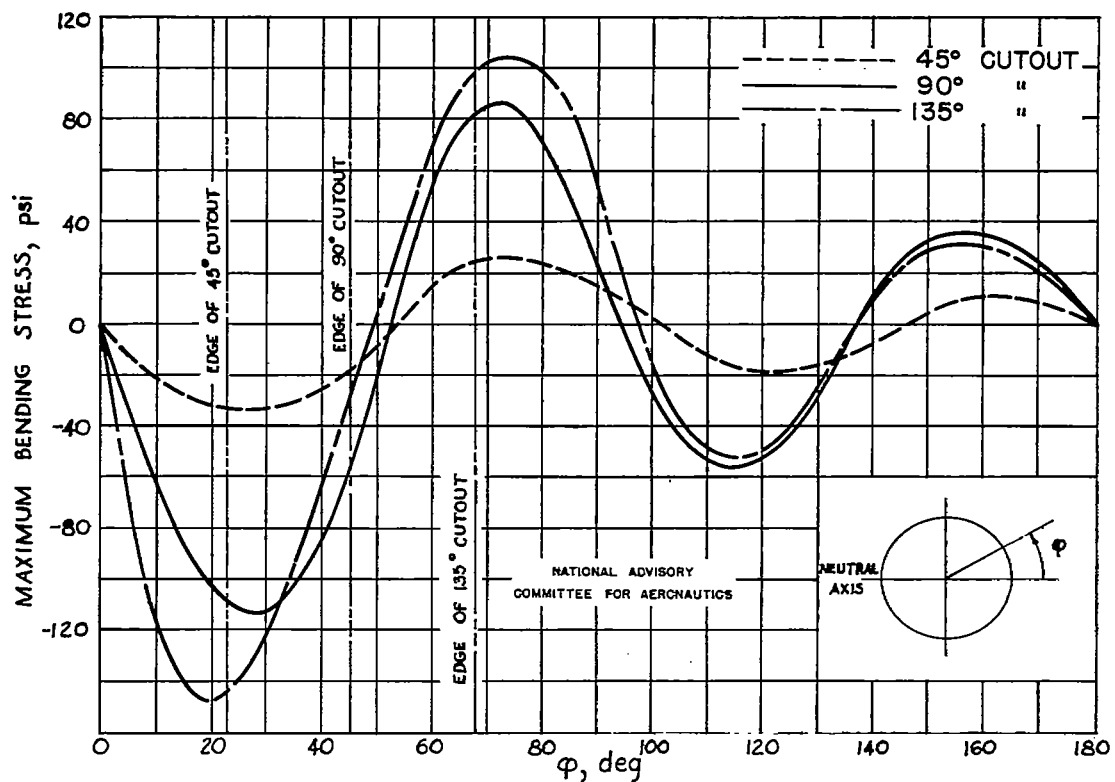


Figure 20.- Comparison of variation of normal strain. Cutout section, 8 stringers, 135° cutout, $M = 20,000$ in-lb.

Figure 21.- Average shear stress distribution in full sections. $M = 20,000$ in-lb.Figure 22.- Bending stress distribution in full rings. $M = 20,000$ in-lb.